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An Economic Attributes Acceptance Sampling Plan With Inspection Errors and Hald Linear Cost Model

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An economic acceptance sampling plan by attributes is developed here when inspection errors are considered. The Hald linear cost model is discussed with gamma prior for the distribution of the fraction of defective items in a lot and the optimal sampling plans can be easily found .

(1) Introduction

In single sampling plans, a random sample of n items is selected from a lot of size N and inspected. As a result of the inspection process, the lot may be either accepted if the number of defective items in the sample is smaller than or equal to some number c ; otherwise it is rejected. In acceptance sampling by attributes, two types of inspection errors are possible; the first occurs when the inspector classifies good item as bad, and the second occurs when the inspector classifies bad item as good. Let

e_1=pr[incorrectly classifying a good item as bad]

e_2=pr[incorrectly classifying bad item as good]

Several authors have studied the economic models for the determination of the optimum single sampling plan by attributes and the effects of inspection errors on acceptance sampling plan, for example, Hald (1981) and Guenther (1984), Greenberg and Stokes (1995) and Balamurali and Kalyonasundaram (1997). The most detailed one is proposed by Guenther (1985). He introduced a new method for the determination of the optimum sampling plan using the Hald linear cost model with a beta prior distribution . Hald (1968) has suggested that the binomial approximated by the poisson and the beta prior be replaced by a gamma prior which closely approximates the desired beta distribution, if the mean of the p-distribution is very small (see Guenther (1971)). So in this paper, we will show the Hald linear cost model with inspection errors and determine the optimum sampling plan with gamma prior for the distribution of the fraction of defective items in a lot.

The general outline of the paper is as follows .In section (2) , the Hald linear cost model with and without inspection errors will be reviewed ,in brief .Section (3) is devoted to introduce a method to determine the optimum sampling plan based on the Hald linear cost model with inspection errors and the gamma prior distribution .Section (4) is devoted to the numerical example .

(2) The Hald Linear Cost Model with and without Inspection Errors

Hald (1981) proposed a model reflects the total costs in terms of money corresponding to lot of quality (p) without inspection errors as follows:

$$h(x, X; N, n, c, p) = \begin{cases} ns_1+xs_2+(N-n)A_1+(X-x)A_2 & \text{if } x\le c \\ ns_1+xs_2+(N-n)R_1+(X-x)R_2 & \text{if } x>c \end{cases} \quad (1)$$

where X and x are the number of defective items in the lot and the sample respectively.

S_1= Cost per item for sampling and testing

S_2= Additional costs for defective item found in the sample

A_1=Cost per item associated with the(N-n) items not inspected in an accepted lot [It is often equal zero]

A_2= Cost associated with defective item which is accepted.

R_1=Cost per item of inspecting the remaining (N-n) items in a rejected lot.

R_2=Additional costs associated with a defective item in the remaining (N-n) items of rejected lot.

If we considered the inspection errors in this model in terms of money, then the total costs of inspection associated with lots of quality (P) will composed of the following terms:

$$h(x, X; N, n, c, e_1, e_2) = \begin{cases} nS_1 + xS_2 + (N-n)A_1 + (X-x)A_2 + A_3(N-n)Pe_2 & \text{if } x \le c \\ nS_1 + xS_2 + (N-n)R_1 + (X-x)R_2 + A_4(N-n)(1-P)e_1 & \text{if } x > c \end{cases} \tag{2}$$

where

A_3=Cost per item for type II errors occurs , A_4=Cost per item for type I errors occurs

e_1= Probability of type I error occurs , e_2 = Probability of type II error occurs,

$A_3(N-n)Pe_2$ = Cost of the number of defective items which is classified as good in the uninspected portion and

$A_4(N-n)(1-P)e_1$= Cost of the number of good items which is classified as defective in the uninspected portion.

If we used poisson distribution as approximation to binomial distribution, then the average cost per lot will be:

$$K(N, n, c, p, e_1, e_2) = nK_s(p) + (N-n)\{K_r(p) - [K_r(p) - K_a(p)]\, p_a + K_E p p_a + A_4 e_1(1 - p - p_a)\} \tag{3}$$

where

$K_s(p)=S_1 + S_2p$, $K_a(p)=A_1+A_2p$, $K_r(p)=R_1+R_2p$,

$K_E=A_3e_2 + A_4e_1$ and P_a is the probability of acceptance under poisson distribution.

Each of the cost function $K_s(p)$, $K_a(p)$, $K_r(p)$ and K_E represents the costs of sampling, acceptance, rejection and errors occurs per item respectively.

(3)A linear Cost Model Plans with Gamma Prior Distribution.

Since P is generally unknown and it varies from lot to lot according to the prior distribution ,f(p),. let f(p) is the gamma distribution with density function.

$$f(P) = \frac{1}{\Gamma a} P^{a-1} k^a e^{-kP} \qquad \text{where} \quad a > 0,\ k > 0 \tag{4}$$

The parameters a and k can be estimated by using m samples of size n. Each sample yields an estimate for the fraction defective (μ_i), i=1,2 ,...... m

$$\text{then} \quad \mu = \frac{1}{m}\sum_{i=1}^{m}\mu_i \quad \text{and} \quad \hat{\sigma}^2 = \frac{1}{m-1}\sum_{i=1}^{m}(\hat{\mu}_i - \mu)^2$$

The moment estimates of a and k are :

$$\hat{a} = \frac{\hat{\mu}^2}{\hat{\sigma}^2} \tag{5}$$

and

$$\hat{k} = \frac{\hat{\mu}}{\hat{\sigma}^2} \tag{6}$$

To get the new average cost per lot over all possible values of P using prior distribution f(P), let $K(N,n,c,e_1)$ be the average of $K(N,n,c,p,e_1,e_2)$ and rewrite (3) as follows:

$$K(N,n,c,p,e_1,e_2) = n(S_1 + S_2P) + (N-n)\{(R_1 + R_2P) + (A_1 - R_1) + (A_2 - R_2)PP_a + K_E PP_a + A_4e_1(1 - P - P_a)\}$$

then

$$K(N,n,c,e_1)$$

$$= nQ_1 + (N-n)\{Q_2 + (A_1 - R_1) + (A_2 - R_2)\frac{a}{k}F' + k_E\frac{a}{k}F' + A_4e_1(1 - \frac{a}{k} - F')\} \tag{7}$$

where $Q_1 = S_1 + S_2\frac{a}{k}$ and $Q_2 = R_1 + R_2\frac{a}{k}$,

and $F' = F(c, a+x, \frac{k}{n+k})$ denotes the negative binomial distribution with parameters a and k

Logically, it would be expected that $S_1 \geq R_1$ and $S_2 \geq R_2$. So, if $S_1=R_1$ and $S_2=R_2$ Guenther (1985) has suggested the following function :

$$R(N,n,c,p) = \frac{K(N,n,c,p) - NQ_2}{Q_3 - Q_2}$$

where $Q_3 = A_1 + A_2\frac{a}{k}$ and $K(N,n,c,p)$ represents the average cost per lot based on the Hald linear cost model without inspection error. So , here we will define the following function :

$$R(N,n,c,e_1) = \frac{K(N,n,c,e_1) - NQ_2}{Q_3 - Q_2}$$

$$\therefore R(N,n,c,e_1) = \frac{n(Q_1-Q_2) + (N-n)\left[A_1 - R_1 + (A_2-R_2)\frac{a}{k}F' + K_E\frac{a}{k}F' + A_4 e_1(1-\frac{a}{k}-F')\right]}{(A_1-R_1)+(A_2-R_2)\frac{a}{k}} \tag{8}$$

If $S_1 > R_1$ or $S_2 > R_2$ or both , then

$$R(N,n,c,e_1) = \frac{K(N,n,c,e_1) - NQ_2}{Q_1 - Q_2}$$

$$\therefore R(N,n,c,e_1) = n + \frac{(N-n)\left[A_1 - R_1 + (A_2-R_2)\frac{a}{k}F' + K_E\frac{a}{k}F' + A_4 e_1(1-\frac{a}{k}-F')\right]}{(S_1-R_1)+(S_2-R_2)\frac{a}{k}} \tag{9}$$

The optimum or Bayeison single sampling plan is defined as the plan minimizing $R(N,n,c,e_1)$ with respect to n and c . The procedure for minimizing $R(N,n,c,e_1)$ is based on determining n which gives the smallest value for $R(N,n,0,e_1)$, then determine n which gives the smallest value for $R(N,n,1,e_1)$, and so on, until the minimum value has been found.
Guenther (1985) gives the cost constants $S_1 = R_1 = 0.01$, $S_2 = R_2 = 0$, $A_1 = 0$, $A_2 = 0.1$, $\hat{\mu} = 0.1583$ and $\hat{\sigma}^2 = 0.01046$ based on 100 lots of size 157 each , he pointed out that the optimum sampling plan is (42, 3) and the minimum average cost per lot equals 1.47 . Here, if we let $A_3 = A_4 = 0.2$, $e_1 = 0.1$, $e_2 = 0.05$.
From (5) and (6), it follows that $\hat{a} = 1.91$ and $\hat{k} = 15.13$ .For fixed C the minimum values of (8) are :

C=0 :R (157,16,0 , 0.1) = 348.74
C=1 :R(157,25,1,0.1) = 277.86
C=2 :R(157, 34 ,2 ,0.1) = 220.98
C=3 :R(157, 42,3,0.1) = 223.02

Then, the Bayesian sampling plan is (34 , 2) and using equation (7), the minimum average cost per lot is K(157,34,2,.1)=2.151 . From the above mentioned it can be seen that, in spite of the inspection errors causes an increasing in the average cost per lot by the amount (2.151 – 1.47 = 0.681) , the sample size tends to be lowest (42 – 34 = 8) .

(4) Conclusion

In this paper , the parameters of single sampling plan (n,c) are determined when inspection errors are considered .The method discussed here is differs in two respects from the method which introduced by Guenther (1985) . First , it uses the Hald linear cost model with inspection errors. Second , it is depends on the poisson approximation to binomial distribution and gamma prior.

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