SOME PROBLEMS OF TESTING EQUALITY BETWEEN TWO INDEPENDENT PROPORTIONS

By
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Part 1: **Introduction**

1.1 Motivation

The comparison of two independent binomial samples is one of the most commonly encountered problems in Statistics. Specifically, we observe x successes out of n observations from population 1 and y successes out of m observations from population 2, and we are interested in testing $p_1 = p_2$, where both are the unknown true successes in the two populations. And alternatively, we are interested in testing $p_1 \neq p_2$. For example, the birth rates in Cairo City and Alexandria City in 2003, or incidences of major malfunctions among newborns both for mothers who were smokers and for mothers who were not.

However, there are numerous methods for testing the null hypothesis that two proportions are equal. And some methods behave poorly in small sample sizes, or when the true proportions are at the extreme values. It is therefore desired for us to obtain the most powerful methods in all possible situations.

1.2 Purpose of the Study

This paper consider a continuation for Amr (A) were we compared the performance of six method to determine which methods for testing equality between two independent proportions are more desirable when the sizes of two samples are either equal or unequal. and the underlying true proportion lies between 0 and 1 and in this paper we will compare another 4 and for simplification purposes we will name the four methods VII, VIII, IX, X. there are three ways to compare two proportions or rates. First is to compare the rate difference (RD); second is to compare the rate ratio-relative risk (RR); and third is to compare the odds ratio (OR). We have chosen four methods (other than the six methods introduced in Amr A) from these categories under various conditions whether with pooling variance, continuity correction or data transformation or not.

To compare the performance of these four methods, we could obtain the corresponding powers of these methods while their type I error is less than or close to the nominal 0.05-level. A SAS program simulation is carried out to calculate both type I errors and powers of these methods. Tables of the results are shown in Appendix A. A sample of the SAS program is shown in Appendix B, for the four methods used.

Powers are calculated for reviewing of the performance of these methods.

Part 2; **REVIEW OF THE LITERATURE**

2. Pooling Varince

In order to derive a test statistic for testing equality between two proportions from independent binomial samples, we will focus first on situations in which both sample sizes are large. Then because sample proportions individually have an approximately normal distribution, the estimator (p_1 - p_2) also has approximately a normal distribution.

Also we would like to consider the performance of the methods when the sample sizes are either small or unequal.

Devore (1995) suggested when the null hypothesis, $p_1 = p_2$, is true, let p denote the common value of p_1 and p_2. Population proportions. Then the standardized variable has approximately a standard normal distribution. However this variable cannot serve as a test statistic because the value of p is unknown. To obtain a test statistic having approximately a standard normal distribution when the null hypothesis is true, p must be estimated from the sample data.

Assuming that $p_1 = p_2 = p$, instead of separate samples of size n and m from two different populations (two different binomial distributions), we really have a single sample of size $(n + m)$ from one population with proportion p.

We will use the estimator of variance of $\ln\left(\frac{\hat{p}_1}{\hat{p}_2}\right)$, $\frac{(1-\bar{p})}{\bar{p}}(\frac{1}{n}+\frac{1}{m})$,for Method VII, and the estimator of variance of $\ln\left(\frac{\hat{p}_1(1-\hat{p}_2)}{\hat{p}_2(1-\hat{p}_1)}\right)$, $\frac{1}{\bar{p}(1-\bar{p})}(\frac{1}{n}+\frac{1}{m})$, for Method IX.

2.2 Continuiy Correction

Yates (1934) first noted that ordinary chi-squared test, χ^2, for testing he null hypohesis, p = 0.5, in a simple dichotomy (i.e. 2x2 tables) seemed to always underestimate the probability of obtaining a given number of successes, the discrepancies being quite large except at the extreme tails. For example, the true probability of obtaining 8 or more successes out of 10 trials is .0547, whereas square root of chi-squared test, χ, gives a value of .0290.

These discrepancies are primarily due to the fact that chi-sqaured test, χ^2, is a continuous distribution, but the binomial distribution it is endeavoring to approximate is discontinuous. By taking half a unit less than the true deviations, for example, 7.5 instead of 8 successes, we may expect to obtain a much closer approximation to the true distribution. This modification is well known as correction for continuity.

Other normal approximations have been developed. For example, Feller (1945) developed a normal approximation for the binomial distribution function, and a similar result obtained by Nicholson (1956) for the hypergeometric distribution.

The continuity correction has been widely discussed in the literature after first introduced by Yats in 1934. Mantel and Greenhouse (1968) supported the use of continuity correction with a two-stage argument, saying:

1. The proper probability model to use in 2x2 table is the one with both sets of marginal totals fixed, which yields the hypergeometric distribution function, and
2. The correction improves probability estimates for the hypergeometric distribution except in pathological cases, such as when the distribution is sufficiently asymmetric.

According to Gart, it is appropriate to apply the commonly used adjustment by simply adding 0.5 to each cell whenever any of the cell frequency equals 0 in a 2x2 contingency table (as shown in Figure 2.1). thus we could avoid the concern when the parameters lie on the boundary of the range (i.e. 0 and 1). We will apply this adjustment to Method VII through Method X. Method VII is logarithm transformation of relative risk with pooling variance, and Method VIII is logarithm transformation of relative risk without pooling variance. Method IX is logarithm transformation of odds ratio with pooling variance, and Method X is logarithm transformation of odds ratio without pooling variance.

Characteristic A	Characteristic B: Present	Characteristic B: Absent	Total
Present	x	n-x	n
Absent	y	m-y	m
Total	(x+y)	(n+m)(x+y)	n+m

Figure 2.1 2x2 contingency table in cell frequencies

We define $p_1 = \frac{x}{n}$ and $\hat{p}_2 = \frac{y}{m}$. Thus we may consider a 2x2 contingency table as the result of two independent binomial samples.

2.3 Data Transformation

Oehlert (2000) stated that the undelrying assumptions for the analysis of variance and (1) independent, (2) normality distributed, and (3) constant variance. These assumptions are important for the validity of some statistical procedures. But in some cases, constant variance is intermediate, and non-constant variance could have a substantial effect on our inferences. The primary tool for dealing with violations of assumptions is *transformation* of the response.

Part 3; METHODS

3.1 Model Assumptions

Let p_1 and p_2 denote the proportions of successes in population 1 and population 2, and we could estimate p_1 and p_2 from sample date, say $p_1 = \frac{x}{n}$ and $p_2 = \frac{y}{m}$, where x and y denote the number of successes, and n and m denoted he number of independent trials in population 1 and population 2 (as shown in Figure 3.1). We will assume that the distribution of x can be taken to be binomial variable with parameters n and p_1, and similarly, y is taken to be a binomial variable with parameters m and p_2 (as shown in figure 3.2).

Population	Outcome		Total
	Success	Failure	
1	x	n-x	n
2	y	m-y	m
Total	(x+y)	(n+m)(x+y)	n+m

Figure 3.1 Binomial distribution table in cell frequencies

Population	Outcome		Total
	Success	Failure	
1	p_1	$1-p_1$	1
2	p_2	$1-p_2$	1

Figure 3.2 Binomial distribution table in cell probabilities

Pirie and Hamdan (1972) described that in 2x2 comparative trials, if only one set of margins, say *n* and *m* is assumed fixed. Thus we could consider this table as the result of two independent binomial samples with parameters p_1 and p_2. This is also called retrospective or prospective studies. We could be interested in the appropriate statistical tests of independence for this contingency table or the equality between two proportions. Chi-squared test is probably now almost universally admitted test for testing the independence of the contingency table. But we will focus on the tests for testing the equality between two independent proportions of 2x2 contingency table in this paper.

Miettinen and Nurminen (1985) stated that comparative analysis of two proportions is a commonly statistical practice in the study of the occurrence of successes in a fixed number of a sequence of independent trials. For example, in case-control study, we would like to compare the proportion of exposure in both case and control group.

Comparative analysis concerns the relative magnitude of the expected proportions, P_1 and P_2. The relative magnitudes are commonly compared thought (1) rate difference (RD), (2) rate ratio (RR), or (3) odds ratio (OR):

$$RD = p_1 - p_2$$

$$RR = \frac{P_1}{P_2}$$

$$OR = \frac{p_1(1-p_2)}{p_2(1-p_1)}$$

For testing the hypotheses:

$$H_0: p_1 = p_2 \text{ vs. } H_1: p_1 \neq p_2$$

This also indicated that testing RD=0, RR=1, OR=1 under the null hypothesis.

3.2 Description of Methods

Zar (1974) indicated that if n, sample size, is very large (say, greater than 25), and neither np_1 nor $n(1-p_1)$ is very small (say, no less than 5), then a normal approximation is available for the binomial test.

3.2.1 Method VII - Log Transformation of Relative Risk with Pooling Variance

In prospective studies, we define that relative risk as the incidence rate (risk) of disease with exposure divided by the incidence rate (risk) of disease without exposure. Figure 3.3 lists the cell frequencies in 2x2 contingency table in prospective studies.

	Disease	Non-Disease	Total
Exposure	x	n-x	n
Non-Exposure	y	m-y	m
Total	(x+y)	(n+m)-(x+y)	n+m

Figure 3.3 2x2 contingency table in prospective studies

The relative risk or risk ratio (RR) is simply the ratio of the two risks,

$$RR = \frac{x/n}{y/m} = \frac{\hat{p}_1}{\hat{p}_2}$$

Values of RR are usually solved from an approach involving log transformation and first-order Taylor series approximation of the variance of $\ln(\frac{\hat{p}_1}{\hat{p}_2})$. By use of the delta method (Anderson, 1958), the estimator of variance of $\ln(\frac{\hat{p}_1}{\hat{p}_2})$ is

$$\frac{(1-\hat{p}_1)}{n\hat{p}_1} + \frac{(1-\hat{p}_2)}{m\hat{p}_2}$$

Under the null hypothesis, $p_1 = p_2 = p$, we could obtain the estimator of variance as

$$\frac{(1-\hat{\bar{p}})}{\hat{\bar{p}}}\left(\frac{1}{n}+\frac{1}{m}\right).$$

where $\hat{\bar{p}} = \frac{n\hat{p}_1 + m\hat{p}_2}{n+m}$

we will call the test statistic derived on the basis of the above estimators as Method VII to be that

$$Z_7 = \frac{\ln\frac{\hat{p}_1}{\hat{p}_2} - \ln RR}{\sqrt{\frac{(1-\hat{\bar{p}}_1)}{\hat{\bar{p}}_1} + \left(\frac{1}{n}+\frac{1}{m}\right)}}$$

3.2.2 Method VIII - Log Transformation of Relative Risk without Pooling Variance

Also we could consider the situation when the estimator of variance of ln (RR) is not pooling. Using the delta method as above, we could obtain the estimator of variance as

$$\frac{(1-\hat{p}_1)}{n\hat{p}_1} + \frac{(1-\hat{p}_2)}{m\hat{p}_2}$$

thus we will have a test statistic referred to as Method VIII as the following

$$Z_8 = \frac{\ln \frac{p_1}{p_2} - \ln RR}{\sqrt{\frac{(1-p_1)}{n p_1} + \frac{(1-p_2)}{m p_2}}}$$

3.2.3 Method IX - Log Transformation of Odds Ratio with Pooling Variance

Odds ratio (OR) is a measure of the degree of inequality between two rates or a measure of association between exposure and outcome. Let p_1 denote the rate at which an event occurs in the first population, and then the odds associated with that event in the first population are $\Omega_1 = p_1/(1-p_1)$. Similarly, the odds associated with the event in the second population are $\Omega_2 = p_2/(1-p_2)$. The odds ratio is simply the ratio of the two odds. We define odds ratio as OR $= \frac{\Omega_1}{\Omega_2}$. Odds ratio is a good approximation to the relative risk. We can apply the delta method to obtain the asymptotic variance of $\ln(O\hat{R})$,

$$[\frac{1}{n p_1 (1-p_1)}] + [\frac{1}{m p_2 (1-p_2)}]$$

Under the null hypothesis, $p_1 = p_2 = p$, we could obtain the estimator of variance as

$$\frac{1}{\bar{p}\left(1-\bar{p}\right)}\left(\frac{1}{n}+\frac{1}{m}\right)$$

where $\bar{p} = \frac{n p_1 + m p_2}{n+m}$

We will refer to the test statistic derived on the basis of the above estimators as Method IX to be that

$$Z_9 = \frac{\ln \frac{p_1(1-p_2)}{(1-p_2)} - \ln OR}{\sqrt{\frac{1}{\bar{p}_1(1-\bar{p}_1)}\left(\frac{1}{n}+\frac{1}{m}\right)}}$$

3.2.4 Method X-log Transformation of Odds Ratio without Pooling Variance

Also we could consider the situation when the estimator of variance of $\ln(\hat{OR})$ is not pooling. Using the delta method, we could obtain the estimator of variance as

$$[\frac{1}{n\,p_1(1-p_1)}]+[\frac{1}{m\,p_2(1-p_2)}],$$

thus we will refer to the method as Method X to be the following

$$Z_{10}=\frac{\ln\frac{p_1(1-p_2)}{p_2(1-p_1)}-\ln OR}{\sqrt{\frac{1}{n\,p_1(1-p_1)}+\frac{1}{m\,p_2(1-p_2)}}}$$

3.3 Simulations

In order to evaluate the performance of these four methods previously mentioned, a simulation program was carried out by using SAS (1990). We apply the random number to generate the random number sequences for the binomial. We consider the situation for both balanced sampling and unbalanced sampling, where the sample size is set equal to 20, 50, 100, and 200, various values of p_1, $p_1 = 0.05$ to 0.95 by 0.1, and the relative risk (RR = $\frac{p_1}{p_2}$) is set equal to 0.25, 0.5, 1.0, 2.0, and 4.0. the computer generates 10,000 repeated samples to calculate both type I errors and the corresponding powers for these four methods by SAS. Tables of the results are shown in Appendix A, the SAS program is shown in Appendix B.

Also, we will apply the continuity correction to Method VII through Method X by simply adding 0.5 to each cell whenever any of the cell frequencies equal 0, or say either p_1 or p_2 is equal zero or one, in a 2x2 contingency table.

Results

Part (4);

To present the performance of these four methods, Table A.1 summarizes the results of the type I errors for possible combinations of the desired sample size ranging from 20 to 200 and the desired underlying probability ranging from 0.05 to 0.95. Note that *type I error* (*alpha*) indicates that the probability of rejecting the null hypothesis when the null hypothesis is true. We would like to have type I error being smaller or closer to the nominal 0.05-level if possible.

Table A.2 to Table A.11 list the results of the performance of power calculations for the desired relative risk (ratio of P_1 and P_2) ranging from 0.25 to 4.0. For example, Table A.2 lists the result of *powers* when two sample sizes are set equal to 20. Note that power indicates that the probability of rejecting the null hypothesis when the null hypothesis is not true. We would like to have methods as powerful as possible while the corresponding type I error is less than or close to the nominal 0.05-level.

As shown in Table A.1 the type I errors of using Method VII and IX are higher than the nominal 0.05-level under various situations. When both sample sizes increase to 200 and the underlying probability is between 0.15 and 0.85, however, type I errors of using Method VII and IX approach toward the nominal 0.05-level. Hence, all four methods are suitable to use when both sample sizes are equal to or greater than 200 and the underlying probability is between 0.15 and 0.85.

Type I error of using Method VII is less than or close to the nominal 0.05-level when one of the sample sizes is small (n=20) and the underlying proability is greater than 0.75, or when both sample sizes are equal to or greater than 50 and the undelrying proability is greater than 0.55 except when n=m=100 and p_1=0.55, and n=100, m=200 and p_1=0.55. Type I error of using Method IX is less than or close to the nominal 0.05-level when one of the sample sizes is small (n=20) and the other sample size is large (m>=50), and the underlying probability is either equal to 0.05 or 0.95. so method VII and IX are not recommended for general use when testing the equality between two independent proportions.

Type I errors of using Method VIII, and X agree well with the nominal 0.05-level. But type I error of using Method VIII tends to be greater than the nominal 0.05-level when one of the sample sizes is much larger than the other and the underlying probability is large. For example, n=20, m=100 and p_1=0.75, type I error of using Method VIII is 0.0615 (shown in Table A.1).

Method VIII and X tend to be less conservative comparing to Method VII and IX for various situations considered in this paper.

Among Method VII, VIII and IX and X, method X tend to be more powerful than Method VIII. For example, when n=m=20, p_1=0.45 and RR = 4.0, powers of using Method VIII and X are, 0.5266, and 0.6516, respectively (shown in Table A.2). also X tend to be more powerful than VIII. For example, when n=20, m=100, p_1=0.35 and RR=0.5, powers of using Method VIII and X are 0.6841 and 0.8384, respectively (shown in Table A.4).

Part (5); Summary and Conclusions

This paper has presented four different methods for testing equality between two independent proportions. The first two methods, VII and VIII, are the log transformation of relative risk with or without pooling variance. The other two methods, IX and X, are the log transformation of odds ratio with or without pooling variance.

To evaluate the performance of these four methods, a SAS simulation was used to compare these four methods with respect to both type I errors and powers for various combinations of sample sizes, underlying probability and relative risk (ratio of two proportions). Based on the results shown in Table A.1, type I errors of using Method VII, and IX are often larger than the nominal 0.05-level for various situations.

Method VII is suitable to use when one of the sample sizes is small (n=20) and the underlying probability is greater than 0.75, or when both sample sizes are equal to or greater than 50 and the underlying probability is greater than 0.55 except when n=m=100 and p_1=0.55, and n=100, m=200 and p_1=0.55. Method IX is suitable to use when one of the sample sizes is small (n=20) and the other sample size is large (m>=50), and the underlying probability is either equal to 0.05 or 0.95. Thus, Method, VII and IX are not recommended for general practice. Type I errors of using Method VIII, and X agree well with the nominal 0.05-level. But when one of the sample sizes is much larger than the other, Method VIII is not suitable to use. When both sample sizes increase to 200, and the underlying probability is either equal to 0.05 or 0.95, Method VII and VIII are not suitable to use. But when both sample sizes increase to 200, and the underlying probability is between 0.15 and 0.85, all four methods are suitable to use.

In general, Method VIII, and X are better than Method VII and IX. Sample sizes would have great effect on both type I errors and powers of those using Method VIII. But unlike Method VIII, Method VI and X tend to be more powerful regardless of the sample sizes. Method VIII and X are recommended for general use when testing the equality between two independent proportions and Method X, log transformation of odds ratio without pooling variance, would have better performance in general.

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Table A.1
Type I Error for Method VII to Method X.

n	m	p_1	VII	VIII	IX	X
20	20	0.05	0.0567	0	0.0567	0
		0.15	0.106	0.0027	0.106	0.0112
		0.25	0.0829	0.0159	0.0829	0.0378
		0.35	0.0713	0.0317	0.0716	0.0433
		0.45	0.0657	0.0354	0.0704	0.0371
		0.55	0.0598	0.038	0.0708	0.0397
		0.65	0.0628	0.0364	0.072	0.0435
		0.75	0.0476	0.0376	0.0745	0.035
		0.85	0.0361	0.0358	0.1048	0.0123
		0.95	0.0023	0.0023	0.0579	0.0001
20	50	0.05	0.0407	0.0065	0.0408	0.0106
		0.15	0.0961	0.0277	0.0848	0.0274
		0.25	0.072	0.033	0.0772	0.0343
		0.35	0.064	0.0419	0.0662	0.0398
		0.45	0.06	0.0416	0.0577	0.0431
		0.55	0.0581	0.0457	0.0554	0.0415
		0.65	0.0554	0.0481	0.0625	0.0386
		0.75	0.0517	0.054	0.0799	0.0356
		0.85	0.0462	0.0619	0.0835	0.0282
		0.95	0.0424	0.0062	0.0402	0.0114
20	100	0.05	0.0117	0.0297	0.0135	0.0282
		0.15	0.0869	0.0319	0.0883	0.0271
		0.25	0.0744	0.0336	0.0682	0.031
		0.35	0.0671	0.0408	0.0619	0.0379
		0.45	0.0596	0.0481	0.0665	0.0438
		0.55	0.0591	0.0571	0.0679	0.0436
		0.65	0.0535	0.0636	0.0658	0.0414
		0.75	0.0542	0.0615	0.0705	0.0332
		0.85	0.0458	0.0853	0.092	0.0283
		0.95	0.0493	0.0047	0.0114	0.0278
20	200	0.05	0.0023	0.0392	0.0036	0.0378
		0.15	0.098	0.0383	0.0837	0.0274
		0.25	0.0831	0.0423	0.079	0.0296
		0.35	0.0667	0.0515	0.0677	0.0425
		0.45	0.0641	0.0573	0.0642	0.0466
		0.55	0.0626	0.0613	0.0684	0.0473
		0.65	0.0599	0.0678	0.0697	0.0405
		0.75	0.0526	0.0721	0.0708	0.0305
		0.85	0.0468	0.1033	0.0882	0.0325
		0.95	0.054	0.0021	0.0035	0.0387
50	50	0.05	0.1158	0.0015	0.112	0.0016
		0.15	0.0789	0.0357	0.075	0.0414
		0.25	0.0641	0.0426	0.0627	0.0472
		0.35	0.0588	0.0477	0.0544	0.0501
		0.45	0.0614	0.0444	0.061	0.0609
		0.55	0.053	0.043	0.053	0.053
		0.65	0.0485	0.0454	0.0516	0.0455
		0.75	0.0503	0.0441	0.0576	0.0441
		0.85	0.0539	0.0414	0.0677	0.0402
		0.95	0.0335	0.0208	0.1201	0.0024

Table A.1 continued
Type I Error for Method VII to Method X.

n	m	p_1	VII	VIII	IX	X
50	100	0.05	0.0965	0.0204	0.0965	0.0208
		0.15	0.0625	0.0386	0.0607	0.038
		0.25	0.0615	0.041	0.0587	0.0442
		0.35	0.057	0.047	0.0553	0.0501
		0.45	0.055	0.0451	0.0551	0.0481
		0.55	0.0538	0.0419	0.0516	0.0448
		0.65	0.0531	0.0471	0.0548	0.0479
		0.75	0.0515	0.0508	0.0585	0.0449
		0.85	0.0507	0.06	0.0654	0.0413
		0.95	0.0311	0.044	0.1002	0.023
50	200	0.05	0.0824	0.0307	0.0826	0.0306
		0.15	0.0658	0.043	0.0635	0.0404
		0.25	0.0597	0.0437	0.0591	0.0449
		0.35	0.0583	0.0499	0.0558	0.0477
		0.45	0.0584	0.0486	0.0578	0.0488
		0.55	0.0526	0.0494	0.0561	0.0485
		0.65	0.0498	0.0484	0.0548	0.0454
		0.75	0.0523	0.0544	0.0616	0.0487
		0.85	0.0494	0.0587	0.0638	0.0413
		0.95	0.0397	0.0655	0.0921	0.0304
100	100	0.05	0.0888	0.0166	0.0774	0.0232
		0.15	0.0555	0.0425	0.0555	0.0433
		0.25	0.0561	0.0467	0.0561	0.0496
		0.35	0.0499	0.0459	0.0499	0.0469
		0.45	0.055	0.048	0.055	0.055
		0.55	0.0559	0.051	0.056	0.0559
		0.65	0.05	0.0477	0.0519	0.0488
		0.75	0.051	0.0479	0.0536	0.0484
		0.85	0.0503	0.0493	0.0579	0.0432
		0.95	0.0447	0.0438	0.0785	0.0258
100	200	0.05	0.0856	0.03	0.0863	0.03
		0.15	0.0606	0.0483	0.0612	0.049
		0.25	0.0596	0.0478	0.0567	0.0513
		0.35	0.051	0.0453	0.0487	0.0466
		0.45	0.0538	0.0455	0.0527	0.0484
		0.55	0.0552	0.0467	0.0539	0.0508
		0.65	0.052	0.0504	0.0528	0.0494
		0.75	0.0522	0.0495	0.054	0.0505
		0.85	0.053	0.0531	0.0601	0.0488
		0.95	0.047	0.0613	0.0872	0.029
200	200	0.05	0.0627	0.0386	0.0627	0.0388
		0.15	0.052	0.0442	0.052	0.0476
		0.25	0.0533	0.0479	0.052	0.0495
		0.35	0.0496	0.0476	0.0496	0.049
		0.45	0.0496	0.0496	0.0496	0.0496
		0.55	0.0468	0.0468	0.0468	0.0468
		0.65	0.0481	0.0466	0.0485	0.0475
		0.75	0.047	0.0459	0.0483	0.0469
		0.85	0.0494	0.0472	0.0523	0.0472
		0.95	0.0461	0.0456	0.0606	0.0361

Table A.2
Power when n=20 and m=20.

p_1	RR	p_2	VII	VIII	IX	X
0.05	0.25	0.2	0.4434	0.026	0.4434	0.0666
	0.5	0.1	0.1432	0.0004	0.1432	0.0012
	1	0.05	0.0573	0.0001	0.0573	0.0002
	2	0.025	0.0533	0	0.0533	0
	4	0.0125	0.0568	0	0.0568	0.0001
0.15	0.25	0.6	0.8979	0.8211	0.8983	0.8471
	0.5	0.3	0.2712	0.0825	0.2712	0.1498
	1	0.15	0.0979	0.0027	0.0979	0.0099
	2	0.075	0.1901	0.0031	0.1901	0.016
	4	0.0375	0.3494	0.0061	0.3494	0.0202
0.25	0.5	0.5	0.4296	0.3023	0.4318	0.335
	1	0.25	0.079	0.0152	0.0479	0.0388
	2	0.125	0.2594	0.0396	0.2594	0.0977
	4	0.0625	0.5254	0.0792	0.5254	0.1709
0.35	0.5	0.7	0.6329	0.5702	0.6651	0.5734
	1	0.35	0.0687	0.0297	0.0693	0.0423
	2	0.75	0.297	0.1313	0.297	0.2035
	4	0.0875	0.6362	0.2805	0.6362	0.4214
0.45	0.5	0.9	0.9076	0.8652	0.9225	0.8826
	1	0.45	0.0667	0.0372	0.0714	0.0389
	2	0.225	0.3853	0.2531	0.3859	0.3027
	4	0.1125	0.7468	0.5266	0.7468	0.6516
0.55	1	0.55	0.0562	0.0338	0.0673	0.0357
	2	0.275	0.4912	0.382	0.4983	0.3999
	4	0.1375	0.8582	0.747	0.8583	0.8002
0.65	1	0.65	0.062	0.0338	0.0673	0.0357
	2	0.325	0.5849	0.5128	0.4983	0.3999
	4	0.1625	0.9312	0.8819	0.8583	0.8002
0.75	1	0.75	0.0522	0.0402	0.0787	0.0375
	2	0.375	0.7042	0.6393	0.7358	0.6449
	4	0.1875	0.9754	0.9588	0.9777	0.9598
0.85	1	0.85	0.0349	0.0346	0.1033	0.0095
	2	0.425	0.8516	0.783	0.8644	0.8031
	4	0.2125	0.9948	0.9921	0.997	0.9921
0.95	1	0.95	0.0018	0.0018	0.0572	0
	2	0.475	0.9566	0.9475	0.9703	0.9315
	4	0.2375	0.9997	0.9993	0.9997	0.9994

Table A.3
Power when n=20 and m=50.

p1	RR	p2	VII	VIII	IX	X
0.05	0.25	0.2	0.6292	0.0037	0.5808	0.0337
	0.5	0.1	0.1608	0.001	0.1501	0.002
	1	0.05	0.0374	0.0055	0.0379	0.0091
	2	0.025	0.0862	0.0112	0.0865	0.0132
	4	0.0125	0.1457	0.012	0.1457	0.0132
0.15	0.25	0.6	0.9794	0.9111	0.969	0.9553
	0.5	0.3	0.3885	0.0611	0.3829	0.1425
	1	0.15	0.0906	0.0271	0.0803	0.0263
	2	0.075	0.1105	0.1216	0.13	0.1501
	4	0.0375	0.2937	0.2455	0.3119	0.2932
0.25	0.5	0.5	0.6011	0.333	0.5376	0.4574
	1	0.25	0.073	0.0356	0.0814	0.033
	2	0.125	0.1885	0.2656	0.2272	0.2549
	4	0.0625	0.4683	0.5375	0.5239	0.5709
0.35	0.5	0.7	0.8372	0.6436	0.7778	0.757
	1	0.35	0.065	0.0407	0.0648	0.0392
	2	0.75	0.2634	0.3713	0.3262	0.3352
	4	0.0875	0.6358	0.731	0.6861	0.7242
0.45	0.5	0.9	0.9849	0.9304	0.9718	0.9748
	1	0.45	0.061	0.0455	0.0598	0.0444
	2	0.225	0.3783	0.5036	0.448	0.4481
	4	0.1125	0.791	0.8722	0.8366	0.8517
0.55	1	0.55	0.0589	0.0436	0.0557	0.041
	2	0.275	0.5005	0.6294	0.5799	0.5667
	4	0.1375	0.9046	0.9473	0.9325	0.9344
0.65	1	0.65	0.0589	0.0491	0.0663	0.0421
	2	0.325	0.6407	0.755	0.7219	0.6986
	4	0.1625	0.9653	0.9824	0.9772	0.9769
0.75	1	0.75	0.0536	0.0556	0.0778	0.0345
	2	0.375	0.7755	0.8703	0.8451	0.8211
	4	0.1875	0.9914	0.9971	0.9945	0.9944
0.85	1	0.85	0.0433	0.0545	0.0767	0.0272
	2	0.425	0.9145	0.9589	0.9521	0.9289
	4	0.2125	0.9995	1	0.9999	0.9999
0.95	1	0.95	0.0429	0.0063	0.0386	0.012
	2	0.475	0.9897	0.997	0.9969	0.9889
	4	0.2375	1	1	1	1

Table A.4
Power when n=20 and m=100.

p_1	RR	p_2	VII	VIII	IX	X
0.05	0.25	0.2	0.6932	0.0002	0.6848	0.0083
	0.5	0.1	0.1589	0.0032	0.1591	0.0021
	1	0.05	0.0095	0.0317	0.0112	0.0303
	2	0.025	0.046	0.0793	0.0482	0.079
	4	0.0125	0.1078	0.1295	0.1087	0.1295
0.15	0.25	0.6	0.9923	0.9435	0.9866	0.9806
	0.5	0.3	0.4698	0.0339	0.4232	0.1512
	1	0.15	0.088	0.0325	0.0903	0.0259
	2	0.075	0.0828	0.2178	0.0957	0.1806
	4	0.0375	0.2358	0.4639	0.2789	0.4429
0.25	0.5	0.5	0.6774	0.3574	0.62	0.5259
	1	0.25	0.0794	0.0372	0.0764	0.0345
	2	0.125	0.1597	0.3333	0.1951	0.3057
	4	0.0625	0.4535	0.6917	0.483	0.6457
0.35	0.5	0.7	0.8895	0.6841	0.8176	0.8348
	1	0.35	0.0698	0.0413	0.0628	0.0391
	2	0.75	0.229	0.4502	0.2995	0.4123
	4	0.0875	0.6371	0.829	0.6787	0.802
0.45	0.5	0.9	0.9948	0.948	0.9781	0.9914
	1	0.45	0.0608	0.0498	0.0671	0.0458
	2	0.225	0.3537	0.5828	0.4564	0.5398
	4	0.1125	0.7998	0.9251	0.8406	0.9138
0.55	1	0.55	0.0568	0.0534	0.0606	0.0417
	2	0.275	0.4952	0.7181	0.6112	0.6596
	4	0.1375	0.9118	0.974	0.9408	0.9704
0.65	1	0.65	0.0551	0.0646	0.0673	0.0423
	2	0.325	0.6523	0.8304	0.7725	0.7744
	4	0.1625	0.9724	0.9938	0.9833	0.9908
0.75	1	0.75	0.0533	0.0687	0.0769	0.0348
	2	0.375	0.7959	0.9256	0.9029	0.8844
	4	0.1875	0.9946	0.9995	0.9979	0.9988
0.85	1	0.85	0.0428	0.0804	0.0871	0.0269
	2	0.425	0.9333	0.9847	0.9781	0.9678
	4	0.2125	0.9999	1	1	1
0.95	1	0.95	0.0505	0.0042	0.0148	0.0317
	2	0.475	0.9943	0.9999	0.9995	0.9979
	4	0.2375	1	1	1	1

Table A.5
Power when n=20 and m=200.

p_1	RR	p_2	VII	VIII	IX	X
0.05	0.25	0.2	0.7421	0.0001	0.7152	0.0017
	0.5	0.1	0.1989	0.0024	0.1963	0.002
	1	0.05	0.0022	0.0385	0.0032	0.0378
	2	0.025	0.0094	0.1281	0.0121	0.1281
	4	0.0125	0.0445	0.2475	0.0462	0.2475
0.15	0.25	0.6	0.997	0.9574	0.9908	0.9883
	0.5	0.3	0.4942	0.0169	0.4537	0.1558
	1	0.15	0.1041	0.04	0.0872	0.0274
	2	0.075	0.0496	0.2473	0.0723	0.2222
	4	0.0375	0.1363	0.5281	0.1853	0.5266
0.25	0.5	0.5	0.7284	0.3732	0.6438	0.561
	1	0.25	0.0798	0.0441	0.0777	0.0304
	2	0.125	0.1253	0.391	0.1772	0.3405
	4	0.0625	0.3595	0.7405	0.4391	0.7186
0.35	0.5	0.7	0.9224	0.7121	0.849	0.8701
	1	0.35	0.0712	0.0464	0.0659	0.0377
	2	0.75	0.2106	0.5133	0.3096	0.4544
	4	0.0875	0.5834	0.8735	0.661	0.8482
0.45	0.5	0.9	0.9964	0.959	0.9796	0.9956
	1	0.45	0.0641	0.0558	0.0654	0.0452
	2	0.225	0.3467	0.6522	0.4766	0.5736
	4	0.1125	0.7867	0.953	0.8466	0.9383
0.55	1	0.55	0.0581	0.005	0.0589	0.0407
	2	0.275	0.4963	0.7714	0.6469	0.6991
	4	0.1375	0.9126	0.9854	0.9498	0.9794
0.65	1	0.65	0.0585	0.0693	0.0703	0.0414
	2	0.325	0.6647	0.8755	0.7997	0.8157
	4	0.1625	0.9742	0.9965	0.9888	0.9956
0.75	1	0.75	0.052	0.0812	0.0812	0.0304
	2	0.375	0.8146	0.9534	0.9174	0.9143
	4	0.1875	0.9954	0.9999	0.9985	0.9996
0.85	1	0.85	0.0412	0.0976	0.0837	0.0274
	2	0.425	0.9444	0.9914	0.9835	0.979
	4	0.2125	0.9999	1	1	1
0.95	1	0.95	0.0491	0.001	0.0027	0.0398
	2	0.475	0.997	1	1	0.9993
	4	0.2375	1	1	1	1

Table A.6
Power when n=50 and m=50.

p_1	RR	p_2	VII	VIII	IX	X
0.05	0.25	0.2	0.7461	0.5545	0.7748	0.5909
	0.5	0.1	0.2632	0.0523	0.234	0.0566
	1	0.05	0.1222	0.002	0.1178	0.0021
	2	0.025	0.1857	0.0227	0.1847	0.0027
	4	0.0125	0.2803	0.0022	0.2794	0.0022
0.15	0.25	0.6	0.999	0.9983	0.9987	0.9985
	0.5	0.3	0.4773	0.4069	0.4765	0.4196
	1	0.15	0.0709	0.0341	0.0687	0.0396
	2	0.075	0.2953	0.1415	0.2704	0.1575
	4	0.0375	0.646	0.3288	0.603	0.3448
0.25	0.5	0.5	0.7567	0.7322	0.7494	0.7405
	1	0.25	0.0584	0.0405	0.0575	0.0445
	2	0.125	0.4213	0.3321	0.4203	0.3423
	4	0.0625	0.8225	0.7185	0.8168	0.7414
0.35	0.5	0.7	0.9521	0.9369	0.9521	0.9521
	1	0.35	0.0573	0.0439	0.0528	0.0469
	2	0.75	0.5486	0.4859	0.5437	0.5028
	4	0.0875	0.9282	0.8958	0.928	0.8998
0.45	0.5	0.9	0.9992	0.9992	0.9992	0.9992
	1	0.45	0.0582	0.0427	0.058	0.058
	2	0.225	0.6934	0.6534	0.6833	0.6598
	4	0.1125	0.9786	0.9697	0.978	0.9715
0.55	1	0.55	0.0533	0.0432	0.0533	0.0533
	2	0.275	0.8216	0.8002	0.8192	0.8181
	4	0.1375	0.9957	0.9939	0.9951	0.9943
0.65	1	0.65	0.0507	0.0483	0.0543	0.0487
	2	0.325	0.9208	0.8935	0.9207	0.9207
	4	0.1625	0.9995	0.9992	0.9994	0.9993
0.75	1	0.75	0.0523	0.0448	0.0602	0.0448
	2	0.375	0.9756	0.9707	0.9757	0.9756
	4	0.1875	1	1	1	1
0.85	1	0.85	0.0539	0.0397	0.0705	0.039
	2	0.425	0.9961	0.9961	0.9964	0.9961
	4	0.2125	1	1	1	1
0.95	1	0.95	0.0355	0.0233	0.1121	0.0022
	2	0.475	1	1	1	1
	4	0.2375	1	1	1	1

Table A.7
Power when n=50 and m=100.

p_1	RR	p_2	VII	VIII	IX	X
0.05	0.25	0.2	0.8526	0.6468	0.8447	0.6981
	0.5	0.1	0.3249	0.0281	0.3249	0.0449
	1	0.05	0.097	0.0217	0.097	0.0222
	2	0.025	0.1339	0.0665	0.1339	0.0795
	4	0.0125	0.2451	0.0888	0.2451	0.1305
0.15	0.25	0.6	1	0.9999	1	1
	0.5	0.3	0.6309	0.4633	0.6073	0.5102
	1	0.15	0.0612	0.039	0.0586	0.0393
	2	0.075	0.2514	0.2877	0.2644	0.2831
	4	0.0375	0.6536	0.6511	0.6551	0.6528
0.25	0.5	0.5	0.8836	0.815	0.8671	0.8501
	1	0.25	0.0628	0.0412	0.0599	0.0459
	2	0.125	0.4321	0.5015	0.4408	0.4738
	4	0.0625	0.8576	0.8869	0.868	0.8822
0.35	0.5	0.7	0.9905	0.9771	0.9856	0.9852
	1	0.35	0.0574	0.0453	0.0556	0.0482
	2	0.75	0.6118	0.6687	0.6307	0.6548
	4	0.0875	0.9636	0.9758	0.9656	0.9711
0.45	0.5	0.9	1	0.9998	0.9999	1
	1	0.45	0.0578	0.0459	0.0593	0.0492
	2	0.225	0.7736	0.8148	0.7933	0.8014
	4	0.1125	0.993	0.9958	0.9935	0.9956
0.55	1	0.55	0.0563	0.0417	0.0534	0.0467
	2	0.275	0.8926	0.918	0.9083	0.9106
	4	0.1375	0.9996	0.9998	0.9996	0.9997
0.65	1	0.65	0.0554	0.0494	0.0586	0.0509
	2	0.325	0.9652	0.9727	0.9724	0.9703
	4	0.1625	1	1	1	1
0.75	1	0.75	0.0537	0.0554	0.0626	0.0488
	2	0.375	0.9933	0.9955	0.9953	0.9934
	4	0.1875	1	1	1	1
0.85	1	0.85	0.0459	0.0532	0.0595	0.0376
	2	0.425	0.9995	0.9998	0.9998	0.9998
	4	0.2125	1	1	1	1
0.95	1	0.95	0.034	0.0422	0.0992	0.0255
	2	0.475	1	1	1	1
	4	0.2375	1	1	1	1

Table A.8
Power when n=50 and m=200.

p_1	RR	p_2	VII	VIII	IX	X
0.05	0.25	0.2	0.9203	0.7113	0.9111	0.777
	0.5	0.1	0.3708	0.009	0.3669	0.0235
	1	0.05	0.088	0.0306	0.0883	0.0305
	2	0.025	0.0963	0.1748	0.0963	0.1748
	4	0.0125	0.2528	0.3451	0.2528	0.3451
0.15	0.25	0.6	1	1	1	1
	0.5	0.3	0.7165	0.5095	0.6956	0.5717
	1	0.15	0.064	0.0391	0.0603	0.0401
	2	0.075	0.2307	0.3931	0.2424	0.3579
	4	0.0375	0.6288	0.7636	0.6368	0.7622
0.25	0.5	0.5	0.9352	0.8683	0.9178	0.9035
	1	0.25	0.0625	0.0455	0.0612	0.0453
	2	0.125	0.4438	0.6084	0.4756	0.5711
	4	0.0625	0.8794	0.9453	0.8852	0.9359
0.35	0.5	0.7	0.9974	0.9899	0.9949	0.995
	1	0.35	0.0559	0.0453	0.0545	0.0463
	2	0.75	0.6458	0.7697	0.6824	0.7425
	4	0.0875	0.9788	0.9922	0.9813	0.9897
0.45	0.5	0.9	1	1	1	1
	1	0.45	0.0568	0.0526	0.0574	0.0506
	2	0.225	0.8116	0.8916	0.8454	0.8747
	4	0.1125	0.9961	0.9988	0.9975	0.9982
0.55	1	0.55	0.0495	0.0479	0.0523	0.0456
	2	0.275	0.9237	0.9629	0.9435	0.9514
	4	0.1375	0.9998	0.9999	0.9999	0.9999
0.65	1	0.65	0.0541	0.0523	0.0576	0.0499
	2	0.325	0.9809	0.9909	0.9878	0.9886
	4	0.1625	1	1	1	1
0.75	1	0.75	0.0518	0.0556	0.0615	0.0486
	2	0.375	0.9973	0.9993	0.9986	0.9984
	4	0.1875	1	1	1	1
0.85	1	0.85	0.0463	0.0573	0.0606	0.0377
	2	0.425	1	1	1	1
	4	0.2125	1	1	1	1
0.95	1	0.95	0.0437	0.0637	0.0894	0.0338
	2	0.475	1	1	1	1
	4	0.2375	1	1	1	1

Table A.9
Power when n=100 and m=100.

p_1	RR	p_2	VII	VIII	IX	X
0.05	0.25	0.2	0.9343	0.9096	0.934	0.9097
	0.5	0.1	0.3337	0.2168	0.3128	0.2272
	1	0.05	0.0918	0.0179	0.079	0.0244
	2	0.025	0.2452	0.0434	0.2238	0.059
	4	0.0125	0.4888	0.0701	0.4438	0.0966
0.15	0.25	0.6	1	1	1	1
	0.5	0.3	0.7407	0.7072	0.7407	0.7194
	1	0.15	0.057	0.043	0.057	0.044
	2	0.075	0.4302	0.3601	0.4294	0.3605
	4	0.0375	0.8511	0.7843	0.8457	0.7893
0.25	0.5	0.5	0.9567	0.955	0.9567	0.9556
	1	0.25	0.0559	0.0479	0.0559	0.0498
	2	0.125	0.65	0.6072	0.65	0.6179
	4	0.0625	0.9734	0.9636	0.9734	0.9637
0.35	0.5	0.7	0.9989	0.9987	0.9989	0.9989
	1	0.35	0.0512	0.0459	0.0512	0.0472
	2	0.75	0.8248	0.8039	0.8248	0.8107
	4	0.0875	0.9975	0.9964	0.9975	0.9967
0.45	0.5	0.9	1	1	1	1
	1	0.45	0.0553	0.0478	0.0553	0.0553
	2	0.225	0.9312	0.9226	0.9312	0.9243
	4	0.1125	1	1	1	1
0.55	1	0.55	0.0531	0.0477	0.0531	0.0501
	2	0.275	0.9837	0.9828	0.9837	0.9978
	4	0.1375	1	1	1	1
0.65	1	0.65	0.0518	0.0492	0.0536	0.0501
	2	0.325	0.9978	0.9962	0.9978	0.9978
	4	0.1625	1	1	1	1
0.75	1	0.75	0.0512	0.0479	0.0557	0.048
	2	0.375	0.9998	0.9998	0.9998	0.9998
	4	0.1875	1	1	1	1
0.85	1	0.85	0.0496	0.0487	0.0572	0.0444
	2	0.425	1	1	1	1
	4	0.2125	1	1	1	1
0.95	1	0.95	0.0423	0.0418	0.0786	0.0255
	2	0.475	1	1	1	1
	4	0.2375	1	1	1	1

Table A.10
Power when n=100 and m=200.

p_1	RR	p_2	VII	VIII	IX	X
0.05	0.25	0.2	0.984	0.9638	0.9837	0.9692
	0.5	0.1	0.4433	0.2413	0.4433	0.2523
	1	0.05	0.0853	0.0285	0.0864	0.0286
	2	0.025	0.2171	0.189	0.2174	0.189
	4	0.0125	0.4921	0.385	0.4921	0.385
0.15	0.25	0.6	1	1	1	1
	0.5	0.3	0.8709	0.8076	0.8571	0.8267
	1	0.15	0.0537	0.0417	0.0527	0.0433
	2	0.075	0.4655	0.5144	0.4823	0.514
	4	0.0375	0.8923	0.912	0.9017	0.912
0.25	0.5	0.5	0.9919	0.9854	0.9902	0.989
	1	0.25	0.0562	0.0483	0.055	0.0489
	2	0.125	0.7252	0.7673	0.7352	0.7625
	4	0.0625	0.9908	0.9944	0.9921	0.9943
0.35	0.5	0.7	1	0.9998	0.9999	0.9999
	1	0.35	0.0525	0.0475	0.0512	0.0487
	2	0.75	0.8947	0.9168	0.9027	0.9119
	4	0.0875	0.9994	0.9997	0.9994	0.9997
0.45	0.5	0.9	1	1	1	1
	1	0.45	0.0556	0.0497	0.0568	0.0509
	2	0.225	0.971	0.9794	0.9752	0.9774
	4	0.1125	1	1	1	1
0.55	1	0.55	0.0533	0.0445	0.0505	0.0472
	2	0.275	0.9963	0.9974	0.997	0.997
	4	0.1375	1	1	1	1
0.65	1	0.65	0.0522	0.0494	0.0537	0.0504
	2	0.325	0.9997	0.9999	0.9999	0.9998
	4	0.1625	1	1	1	1
0.75	1	0.75	0.0524	0.0513	0.0555	0.0496
	2	0.375	1	1	1	1
	4	0.1875	1	1	1	1
0.85	1	0.85	0.0475	0.0472	0.0547	0.0424
	2	0.425	1	1	1	1
	4	0.2125	1	1	1	1
0.95	1	0.95	0.0455	0.0596	0.0848	0.0308
	2	0.475	1	1	1	1
	4	0.2375	1	1	1	1

Table A.11
Power when n=200 and m=200.

p_1	RR	p_2	VII	VIII	IX	X
0.05	0.25	0.2	0.9986	0.9978	0.9986	0.9982
	0.5	0.1	0.5158	0.4613	0.5158	0.4738
	1	0.05	0.0664	0.0382	0.0664	0.0382
	2	0.025	0.3149	0.2053	0.3149	0.2062
	4	0.0125	0.6741	0.4829	0.6741	0.4921
0.15	0.25	0.6	1	1	1	1
	0.5	0.3	0.9538	0.9488	0.9533	0.9502
	1	0.15	0.0521	0.0441	0.0521	0.0471
	2	0.075	0.6862	0.6516	0.6862	0.6653
	4	0.0375	0.9839	0.9776	0.9839	0.9794
0.25	0.5	0.5	0.9995	0.9995	0.995	0.995
	1	0.25	0.0521	0.0478	0.0515	0.0496
	2	0.125	0.9046	0.8929	0.9046	0.8971
	4	0.0625	0.9998	0.9998	0.9998	0.9998
0.35	0.5	0.7	1	1	1	1
	1	0.35	0.0505	0.0479	0.0505	0.0497
	2	0.75	0.981	0.9788	0.9805	0.9795
	4	0.0875	1	1	1	1
0.45	0.5	0.9	1	1	1	1
	1	0.45	0.0518	0.0518	0.0518	0.0518
	2	0.225	0.9981	0.998	0.9981	0.998
	4	0.1125	1	1	1	1
0.55	1	0.55	0.0477	0.0477	0.0477	0.0477
	2	0.275	0.9998	0.9998	0.9998	0.9998
	4	0.1375	1	1	1	1
0.65	1	0.65	0.0525	0.0509	0.0529	0.0519
	2	0.325	1	1	1	1
	4	0.1625	1	1	1	1
0.75	1	0.75	0.0508	0.0487	0.0516	0.0506
	2	0.375	1	1	1	1
	4	0.1875	1	1	1	1
0.85	1	0.85	0.049	0.0467	0.0528	0.0467
	2	0.425	1	1	1	1
	4	0.2125	1	1	1	1
0.95	1	0.95	0.0495	0.0486	0.062	0.0382
	2	0.475	1	1	1	1
	4	0.2375	1	1	1	1

APPENDIX B
Samples of SAS programs

```
/*program used for calculated Type I error*/
option ps=70 ls=70;
data tmp1;
Za=1.96;
nsimul=10000;
do n=20,50,100,200;
do m=20,50,100,200;
do p=0.05to0.95by0.1;

sum1=0;
sum2=0;
sum3=0;
sum4=0;
sum5=0;
sum6=0;

do i=1 to nsimul;
x1=ranbin(1234567, n, p);
x2=ranbin(1234567, m, p);
p1=x1/n;
p2=x2/m;
p_bar=(n*p1 +m*p2)/(n+m);
q_bar=1-p_bar;
pq=p_bar*q_bar;
p1q1=p1*(1-p1);
p2q2=p2*(1-p2);

/*step for Method VII:
Log transformation of Relative Risk with pooling variance*/
if (x1 NE 0 and x2 NE 0 and x1 NE n and x2 NE m) then do;
p1=x1/n; ,
p2=x2/m;
p_bar=(n*p1 +m*p2)/(n+m);
q_bar=1-p_bar;
z7=(log(p1)-log(p2))/(sqrt((q-bar/p_bar)*((1/n)+(1/m))));
if abs(z7) >= Za then sum7=sum7+1; end;

else do;
p1=(x1+0.5)/(n+1);
p2=(x2+0.5)/(m+1);
```

```
p_bar=((n+1 )*p1 +(m+1 )*p2)/(n+m+2);
q_bar=1-p_bar;
z7=(log(p1)-log(p2))/(sqrt((q_bar/p_bar)*((1/(n+1))+(1/(m+1)))));
if abs(z7) >= Za then sum7=sum7+1; end;

/* step for Method VIII:
Log transformation of Relative Risk without pooling variance*/

if(x1 NE 0 and x2 NE 0 and x1 NE n and x2 NE m) then do;
p1=x1/n;
p2=x2/m;
p_bar=(n*p1 +m*p2)/(n+m);
q_bar=1-p_bar;
z8=((log(p1 )-log(p2)))/(sqrt(((1 -p1 )/(n*p1 ))+((1 -p2)/(m*p2))));
if abs(z8) >= Za then sum8=sum8+1;end;

else do;
p1=(x1+0.5)/(n+1);
p2=(x2+0.5)/(m+1);
p_bar=((n+1 )*p1 +(m+1 )*p2)/(n+m+2);
q_bar=1-p_bar;
z8=((log(p1)-log(p2)))/(sqrt(((1-p1)/((n+1)*p1))+((1-p2)/((m+1)*p2))));
if abs(z8) >= Za then sum8=sum8+1 ;end;

/*step for Method IX:
Log transformation of Odds Ratio with pooling variance*/

if(x1 NE 0 and x2 NE 0 and x1 NE n and x2 NE m) then do;
p1=x1/n;
p2=x2/m;
p_bar=(n*p1 +m*p2)/(n+m);
q_bar=1-p_bar;
q1=1-p1;
q2=1-p2;
z9=(log(p1)-log(p2)+log(q2)-
log(q1))/(sqrt((1/((p_bar)*(q_bar)))*((1/n)+(1/m))));
if abs(z9) >= Za then sum9=sum9+1 ;end;

else do;
p1=(x1+0.5)/(n+1);
p2=(x2+0.5)/(m+1);
p_bar=((n+1 )*p1 +(m+1 )*p2)/(n+m+2);
q_bar=1-p_bar;
```

```
q1=1-p1;
q2=1-p2;
z9=(log(p1)-log(p2)+log(q2)-
log(q1))/(sqrt((1/((p_bar)*(q_bar)))*((1/(n+1))+(1/(m+1)))));
if abs(z9) >= Za then sum9=sum9+1 ;end;

/*step for Method X:
Log transformation of Odds Ratio without pooling variance*/

if (x1 NE 0 and x2 NE 0 and x1 NE n and x2 NE m) then do;
p1=x1/n;
p2=x2/m;
p_bar=(n*p1 +m*p2)/(n+m);
q_bar=1-p_bar;
q1=1-p1;
q2=1-p2;
z10=(log(p1)-log(p2)+log(q2)-
log(q1))/(sqrt((1/(n*(p1)*(q1)))+(1/(m*(p2)*(q2)))));
if abs(z10)>=Za then sum10=sum10+1;end;

else do;
p1=(x1+0.5)/(n+1);
p2=(x2+0.5)/(m+1);
p_bar=((n+1 )*p1 +(m+1 )*p2)/(n+m+2);
q_bar=1-p_bar;

q1=1-p1;
q2=1-p2;
z10=(log(p1)-log(p2)+log(q2)-
log(q1 ))/(sqrt((1/((n+1 )*(p1 )*(q1 )))+(1/((m+1 )*(p2)*(q2)))));
if abs(z10)>= Za then sum10=sum10+1;end;

rejp7=sum7/nsimul;
rejp8=sum8/nsinul;
rbjp9=sum9/nsimul;
rejp10=sum10/nsimul;
end; /*end of simulation*/
output;
end;/*end of p*/
end;/*end of m*/ end;/*end of n*/
run;
data all(keep=n m p rejp7 rejp8 rejp9 rejp10);
merge tmp1 tmp2;
```

```
where n le m;
run;

proc export data=all outfile=Amr excelb.csv' dbms=csv replace;
run;

proc print data=all label;
rejp7='z7' rejp8='z8' rejp9='z9' rejp10='z10';
format rejp7 rejp8 rejp9 rejp10 5.4;
run;

/*Program used for calculated Power*/
option ps=70 ls=70;

data a;
do p=0.05 to 0.95 by. 1;
do r=0.25,5, 1, 2,4;
do pp=p/r;
if pp < 1 then pp=pp;
else pp=.;
output;
end;
end;
end;
run;

data b;
set a;
if pp ne .;
run;

data tmp1;
set b;
Za=1.96;
nsimul=10000;
do n=200 /*sample size: 20,50,100,200*/;
do m=200/*sample size: 20,50,100,200*/;

sum1=0;
sum2=0;
sum3=0;
sum4=0;
sum5=0;
sum6=0;
```

```
do i=1 to nsimul;
x1=ranbin(1234567, n, p);
x2=ranbin(1234567, m, pp);
p1=x1/n;
p2=x2/m;
p_bar=(n*p1 +m*p2)/(n+m);
q_bar=1-p_bar;
pq=p_bar*q_bar;
p1q1=p1*(1-p1);
p2q2=p2*(1-p2);

/*step for Method VII:
Log transformation of Relative Risk with pooling variance*/

if (x1 NE 0 and x2 NE 0 and x1 NE n and x2 NE m) then do;
p1=x1/n;
p2=x2/m;
p_bar=(n*p1 +m*p2)/(n+m);
q_bar=1-p_bar;
z7=(log(p1)-log(p2))/(sqrt((q_bar/p_bar)*((1/n)+(1/m))));
if abs(z7) >= Za then sum7=sum7+1; end;

else do;
p1=(x1+0.5)/(n+1);
p2=(x2+0.5)/(m+1);
p_bar=((n+1 )*p1 +(m+1 )*p2)/(n+m+2);
q_bar=1-p_bar;
z7=(log(p1)-log(p2))/(sqrt((q_bar/p_bar)*((1/(n+1))+(1/(m+1)))));
if abs(z7) >= Za then sum7=sum7+1; end;
/* step for Method VIII:
Log transformation of Relative Risk with pooling variance*/

if (x1 NE 0 and x2 NE 0 and x1 NE n and x2 NE m) then do;
p1=x1/n;
p2=x2/m;
p_bar=(n*p1 +m*p2)/(n+m);
q_bar=1-p_bar;
z8=((log(p1)-log(p2)))/(sqrt(((1-p1)/(n*p1))+((1-p2)/(m*p2))));
if abs (z8) >= Za then sum8=sum8+1 ;end;

else do;
p1=(x1+0.5)/(n+1);
p2=(x2+0.5)/(m+1);
```

```
p_bar=((n+1 )*p1 +(m+1 )*p2)/(n+m+2);
q_bar=1-p_.bar;
z8=((log(p1)-log(p2)))/(sqrt(((1-p1)/((n+1)*p1))+((1-p2)/((m+1)*p2))));
if abs(z8) >= Za then sum8=sum8+1 ;end;

/*step for Method IX:
Log transformation of Odds Ratio with pooling variance*/

if (x1 NE 0 and x2 NE 0 and x1 NE n and x2 NE m) then do;
p1=x1/n;
p2=x2/m;
p_bar=(n*p1 +m*p2)/(n+m);
q_bar=1-p_bar;
q1=1-p1;
q2=1-p2;
z9=(log(p1)-log(p2)+log(q2)-log(q1))/(sqrt((1/((p_bar)*(q_bar)))*((1/n)+
t-(1/m))));
if abs(z9) >= Za then sum9=sum9+1 ;end;
else do;
p1=(x1+0.5)/(n+1);
p2=(x2+0.5)/(m+1);
p_bar=((n+1 )*p1 +(m+1 )*p2)/(n+m+2);
q_bar=1-p_bar;
q1=1-p1;
q2=1-p2;
z9=(log(p1 )-log(p2)+log(q2)-
log(q1)y(sqrt((1/((p_bar)*(q_bar)))*((1/(n+1))+(1/(m+1)))));
if abs(z9) >= Za then sum9=sum9+1 ;end;

/*step for Method X:
Log transformation of Odds Ratio without pooling variance*/

if (x1 NE 0 and x2 NE 0 and xl NE n and x2 NE m) then do;
p1=x1/n;
p2=x2/m;
p_bar=(n*p1 +m*p2)/(n+m);
q_bar=1-p_bar;
q1=1-p1;
q2=1-p2;
z10=(log(p1)-log(p2)+log(q2)-
log(q1))/(sqrt((1/(n*(p1)*(q1)))+(1/(m*(p2)*(q2)))));
if abs(z10) >=Za then sum10=sum10+1;end;
```

```
else do;
p1=(x1+0.5)/(n+1);
p2=(x2+0.5)/(m+1);
p_bar=((n+1 )*p1 +(m+1 )*p2)/(n+m+2);
q_bar=1-p_bar;
q1=1-p1;
q2=1-p2;
z10=(log(p1.)-log(p2)+log(q2)-
log(q1 ))/(sqrt((1/((n+1 )*(p1 )*(q1 )))+(1 /((m+1 )*(p2)*(q2)))));
if abs(z10)>=Za then sum10=sum10+1;end;

rejp7=sum7/nsimul;
rejp8=sum8/nsimul;
rejp9=sum9/nsimul;
rejp10=sum10/nsimul;

end; /*end of simulation*/
output;
end; /*end of m*/
end; /*end of n*/
run;

data all(keep=n m p r pp rejp7 rejp8 rejp9 rejp10);
merge tmp1 tmp2;
run;

proc export data=all outfile='Amr powern200m200.csv' dbms=csv
replace;
run;
proc print data=all label;
rejp7='z7' rejp8='z8'rejp9='z9' rejp10='z10';
format rejp 7 rejp8 rejp9 rejp10 5.4;
run;
```

Appendix C
Derivation of variances

Method VII & Method VIII

Know that $RR = \frac{p_1}{p_2}$; let $f(p_1) = \ln p_1$, then $[f'(p_1)]^2 = \frac{1}{p_1^2}$

Using the delta method,

$$\hat{var}(\ln RR) = \hat{var}\left(\ln\left(\frac{\hat{p}_1}{\hat{p}_2}\right)\right) = \hat{var}[\ln(\hat{p}_1) - \ln(\hat{p}_2)]$$

$$= \hat{var}[\ln(\hat{p}_1)] + \hat{var}[\ln(\hat{p}_2)]$$

$$= [f'(\mu)]^2\, \hat{var}(\hat{p}_1) + [f'(\mu)]^2\, \hat{var}(\hat{p}_2)$$

$$= \frac{1}{\hat{p}_1^2}\left(\frac{\hat{p}_1(1-\hat{p}_1)}{n}\right) + \frac{1}{\hat{p}_2^2}\left(\frac{\hat{p}_2(1-\hat{p}_2)}{m}\right)$$

$$= \frac{1-\hat{p}_1}{n\hat{p}_1} + \frac{1-\hat{p}_2}{m\hat{p}_2}$$

Under the null hypothesis: $p_1 = p_2 = p$, where $\bar{p} = \frac{n\hat{p}_1 + m\hat{p}_2}{n+m}$,

$$\hat{var}(\ln RR) = \frac{1-\bar{p}}{n\bar{p}} + \frac{1-\bar{p}}{m\bar{p}}$$

$$= \frac{m(1-\bar{\hat{p}}_1)}{nm\bar{\hat{p}}} + \frac{n(1-\bar{\hat{p}}_2)}{nm\bar{p}}$$

$$= \frac{(n+m)(1-\bar{p})}{nm\bar{p}}$$

$$= \frac{(1-\bar{\hat{p}})}{\bar{p}}\left(\frac{1}{n} + \frac{1}{m}\right)$$

Method IX & Method X

Know that $(\dot{OR}) = \frac{p_1(1-p_2)}{p_2(1-p_1)}$

Let $f(p_1) = \ln\frac{p_1}{c-p_1} = \ln p_1 - \ln(c-p_1)$, then $[f'(p_1)]^2 = \left(\frac{1}{p_1} + \frac{1}{c-p_1}\right)^2 = \left(\frac{c}{(c-p_1)p_1}\right)^2$

Using the delta method,

$$\hat{\text{var}}(\ln \hat{OR}) = \hat{\text{var}}\left(\ln\left(\frac{\hat{p}_1(1-\hat{p}_2)}{\hat{p}_2(1-\hat{p}_1)}\right)\right) = \hat{\text{var}}\left[\ln\left(\frac{\hat{p}_1}{1-\hat{p}_1}\right) + \ln\left(\frac{1-\hat{p}_2}{\hat{p}_2}\right)\right]$$

$$= \hat{\text{var}}\left[\ln\left(\frac{\hat{p}_1}{1-\hat{p}_1}\right) + \ln\left(\frac{\hat{p}_2}{1-\hat{p}_2}\right)\right]$$

$$= [f'(\mu)]^2\, \hat{\text{var}}(\hat{p}_1) + [f'(\mu)]^2\, \hat{\text{var}}(\hat{p}_2)$$

$$= \left(\frac{1}{(1-\hat{p}_1)\hat{p}_1}\right)^2 \left(\frac{\hat{p}_1(1-\hat{p}_1)}{n}\right) + \left(\frac{1}{(1-\hat{p}_2)\hat{p}_2}\right)^2 \left(\frac{\hat{p}_2(1-\hat{p}_2)}{m}\right)$$

$$= \frac{1}{n\hat{p}_1(1-\hat{p}_1)} + \frac{1}{m\hat{p}_2(1-\hat{p}_2)}$$

Under the null hypothesis: $p_1 = p_2 = p$ where $\hat{\bar{p}} = \frac{n\hat{p}_1 + m\hat{p}_2}{n+m}$,

$$\hat{\text{var}}(\ln \hat{OR}) = \frac{1}{n\hat{p}_1(1-\hat{p}_1)} + \frac{1}{m\hat{p}_2(1-\hat{p}_2)}$$

$$= \frac{1}{n\hat{\bar{p}}(1-\hat{\bar{p}})} + \frac{1}{m\hat{\bar{p}}(1-\hat{\bar{p}})}$$

$$= \left(\frac{1}{\bar{p}(1-\bar{p})}\right)\left(\frac{1}{n} + \frac{1}{m}\right)$$

ABSTRACT

This paper emphasizes on the discussion of four different methods for testing equality between two independent proportions.

The fist two methods, VII and VIII, are the log transformation of relative risk with or without pooling variance. The other two methods, IX and X, are the log transformation of odds ratio with or without pooling variance. To evaluate the performance of these four methods, a SAS simulation was carried out to compare these four methods with respect to both type I errors and powers for various combinations of sample sizes, underlying probability and relative risk (ratio of two proportions). On the basis of the results found in the simulation, Method VIII and X are recommended for general use when testing the quality between two independent proportions. Method X, log transformation of odds ratio without pooling variance, would have better performance in general.