هل يمكن إعتبـار الاقتصـاد المصرى مستقرا من الناحية الديناميكية ؟

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باستخدام نموذج ديناميكى قصير الأجل للاقتصاد المصرى يتكون من أربع معادلات عن الفترة من ١٩٦٠ حتى ١٩٧٥ ، وبعد تقدير معلماته بطريقة المربعات الصغرى ثلاثية المراحل three - stage least squares method واشتقاق الشكل الموجز له reduced form ، تم فحص الجوانب العشوائية وغير العشوائية لقضية الاستقرار التى تعتبر ذات أهمية بالغة لصانعى السياسة فى مصر، كما تم فحص المسار الزمنى الذى تأخذه المتغيرات الداخلية إذا ما حدث أن زاد أحد المتغيرات الخارجية بوحدة نقدية واحدة فى إحدى السنوات .

١ — النموذج الهيكلى :

ويتألف النموذج من معادلة للاستهلاك ومعادلة للاستثمار ومعادلة للسوق النقدى هذا بالاضافة إلى المتطابقة المعروفة الخاصة بالدخل القومى [أنظر المعادلات من (١) الى (٤) على التوالى] . وكما يبدو فالنموذج يهمل جانب العرض إذ لا يشتمل على معادلة للانتاج أو معادلة للعمل . ومن ثم ، فان التغيرات فى الدخل القومى يتم شرحها وفقا لما انتهجه أتباع كينز ، أى بدلالة الطلب الفعال وحده مع تجاهل القيود التى تفرضها كمية ونوعية المتاح من موارد بشرية ومادية .

ويضم النموذج أربعة متغيرات داخلية endogenous variables هى الانفاق الاستهلاكى C_t ، والإنفاق الاستثمارى I_t ، ومعدل الفائدة R_t ، والدخل القومى Y_t ، حيث يشير المذيل t فى جميعها إلى الزمن . هذا بالاضافة إلى سبعة متغيرات محددة سلفاً predetermined variables : إثنان منهم يمثلان قيم

متغيرات داخلية فى السنة السابقة وهما الانفاق الاستهلاكى فى السنة السابقة C_{t-i} والانفاق الاستثمارى فى السنة السابقة I_{t-i} ، أما الخمسة الباقــون فيمثلون متغيرات خارجية هى صافى الصادرات $(X-M)_t$ ، وعدد السكان N_t ، والعرض النقدى G_t ، والواحد الصحيح ليمثل ثوابت المعادلات الاربع .

وتؤلف معادلة الاستهلاك مع معادلة الاستثمار بالاضافة إلى المتطابقة المحاسبية الخاصة بالدخل القومى ما يسمى « بمعادلة الادخار — الاستثمار » IS equation ، وهى المعادلة الخاصة بالتوازن فى السوق السلعى . أما الشق الآخر للنموذج الكينزى المعروف باسم « معادلة السيولة — العرض النقدى » LM-equation ، وهى المعادلة الخاصة بالتوازن فى السوق النقدى ، فتمثله المعادلة رقم (٣) التى تعتبر دالة كينزية غير عادية للطلب النقدى . ولقــد تم الحصول على هذه الدالة (مفترضين أن المرونة الدخلية للطلب على النقود هى الواحد الصحيح ، وهو ما نعتقد أنه الحال فى مصر) بتحويل الدالة الأصلية لوغاريتميا ثم إستخدام التقريب الخطى ووضع معدل الفائدة على اليسار .

واقد إستخدمت فى تقدير معلمات النموذج بيانات سنوية تغطى الفترة من ١٩٦٠ حتى ١٩٧٥ مقاسة بملايين الجنيهات المصرية ومقومة بأسعار ١٩٧٠ باستثناء معدل الفائدة بالطبع (أنظر الملحق) . وتظهر المعادلات (٥) إلى (٨) النموذج بعد تقدير معلماته باستخدام طريقة المربعات ثلاثية المراحل . وتفعل المعادلات (٩) إلى (١٢) نفس الشيء ولكن فى صورة مصفوفات .

٢ — الشكل الموجز وقضية الاستقرار :

ويتطلب التعرف على مدى إستقرار الاقتصاد المصرى كما يمثله النموذج الهيكلى أن نكتب النموذج فى شكله الموجز وذلك بحــل النموذج الهيكلى للمتغيرات

الداخلية ، أى التعبير عن المتغيرات الداخلية بدلالة المتغيرات المحددة سلفاً وعددها سبعة كما ذكرنا سابقاً . وتمثل المعادلة (١٣) الشكل الموجز للنموذج الهيكلى .

$$Z_t = DZ_{t-i} + EX_t + V_t \quad (13)$$

وتمثل المعادلتان (١٤) ، (١٥) مصفوفات المعاملات المحسوبة D و E .

هذا ولدراسة الخواص الديناميكية الداخلية للنموذج المقترح يتطلب الأمر إعادة كتابة المعادلة السابقة بحيث تصبح :

$$Z_t = D^t \; Z_o + \sum_{i=o}^{t-i} D^i \; EX_{t-i} + \sum_{i=o}^{t-i} D^i \; V_{t-i} \quad (22)$$

وتمثل هذه المعادلة المسار الزمنى للمتغيرات الداخلية Z_t . ويتضح منها أن هذا المسار يعتمد على ثلاثة عناصر هى :

١ — الأوضاع المبدئية للمتغيرات وتمثلها Z_o .

٢ — المسار الزمنى للمتغيرات الخارجية وتمثلها X_{t-i} .

٣ — المسار الزمنى للمتغيرات العشوائية وتمثلها V_{t-i} .

ومن ثم ، يمكن لأى من هذه العناصر الثلاث التى يحدث تقلبات فى Z_t . ولكن إهتمامنا فى هذا البحث سوف يوجه إلى دراسة الخواص الديناميكية الداخلية للنموذج inherent dynamic properties ونقصد بها خصائص المسار الزمنى الذى تأخذه المتغيرات الداخلية عقب الانتقال المبدئى من وضع توازنى دون أن يصاحب ذلك تغيرات أخرى فى المتغيرات الخارجية أو العشوائية . أى أن التركيز على الخواص الديناميكية التى يمكن إرجاعها إلى العنصر الأول فى المعادلة (٢٢) وهو $D^t \; Z_o$. ويتطلب ذلك الافتراض بأن كلا من X_{t-i} و V_{t-i}

يساوى الصفر لجميع قيم i بحيث تصبح المعادلة (٢٢) كما يلى :

$$Z_t = D^t \quad Z_0 \tag{23}$$

وتوصف الخواص الديناميكية التى يسببها العنصر $Z_0 \quad D^t$ بأنها داخلية النشأة لارتباطها بجذور المصفوفة D وبالتالى بهيكل النموذج .

وطبقا لاحدى النظريات الشهيرة (Zurmuhl, 1958, P. 181) يمكن التعبير عن أية مصفوفة مربعة مثل D ، ليس لها جذور غير صفرية متعددة ورتبتها N ، كمجموع N من المصفوفات كل منها برتبة واحد rank of one ومن ثم ، يمكن إعادة كتابة المعادلة (٢٣) كما يلى :

$$Z_t = \left(\sum_{n=1}^{N} y_n^t \quad R_n \right) Z_0 \tag{26}$$

وفى هذه المعادلة تمثل λ_n جذور المعادلة D ، كما تمثل المصفوفات R_n وكل منها برتبة واحد حاصل ضرب العمود المتميز رقم n فى الصف المتميز رقم n

the product of the n^{th} characteristic column by the n^{th} characteristic row.

وتساعد المعادلة (٢٦) فى توضيح الكيفية التى يساهم بها العنصر $Z_0 \quad D^t$ فى المسار الزمنى للمتغيرات الداخلية Z_t ، فهذه المتغيرات هى دالة خطية فى جذور المصفوفة (١) . ويشترط لاستقرار هذا المسار الزمنى أن تكون القيمة المطلقة

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(١) تحدد طبيعة هذه الجذور سرعة الاقتراب من وضع الاستقرار . وما إذا كان الاقتراب تدريجيا أو متقلبا . كما أنها تساعد فى التأكد من دقة التمثيل الديناميكى للنموذج ، فإذا كانت الجذور المحسوبة أكبر من واحد بينما الوضع الفعلى يعتبر مستقرا فلا يمكن قبول النموذج كتمثيل للواقع .

الكل من هذه الجذور أقل من الواحد الصحيح [أنظر المعادلة (٢٧)] .

فاذا كانت جميع الجذور موجبة positive (وأقل من الواحد) فان المسار الزمنى المتغيرات الداخلية سوف يتناقص بالتدريج حتى يعود إلى وضع الاستقرار steady state . أما إذا كانت بعض الجذور سالبة (وجميعها أقل من الواحد)، فان المسار الزمنى سوف يتقلب فى هذه الحالة ولكن هذه التقلبات تتضاءل شيئا فشيئا ليقترب فى النهاية من وضع الاستقرار

(Baumol, 1959, PP. 197-201)

وبتحديد جـــذور المصفوفة D فى نموذج الاقتصاد المصرى كما توضحها المعادلة (١٤) ، نجد أن هذه الجذور كالآتى :

$$\lambda_1 = .842$$

$$\lambda_2 = -2.247$$

وكما نرى فان القيمة المطلقة للجذر الثانى أكبر من الواحد الصحيح ، ومن ثم يكون النموذج غير مستقر ويوضح تقلبات تتزايد حدتها بانقضاء الزمن explosive oscillations

٣ — الجوانب العشوائية فى تحليل الاستقرار :

لما كانت جذور المصفوفة D نتاج عملية تقدير احصائى فانها عرضة لخطأ المعاينة ويتحتم بالتالى أن نحسب لها فترة ثقة ، الامر الذى يتطلب ايجاد تباينها . وطبقا لما أوضحه Theil and Boot (1962). ، فان تباين القيمة المطلقة للجذر λ_n تحسب كالآتى :

$$S^2_{|\lambda_n|} = K'_n \; W_o \; K_n \qquad (31)$$

حيث K_n تمثل المتجه العمودى الناتج عن وضع أعمدة المصفوفة H_n أسفل بعضها البعض وقسمتها على مجموع العناصر القطرية فى H_n . وتمثل المصفوفة H_n المصفوفة المصاحبة للمصفوفة $D-\lambda I$. أما W_0 فهى مصفوفة التباين والتغاير لعناصر D.

وبتقدير تباين كل من $|\lambda_1|$ ، $|\lambda_2|$ نجد أنهما كما يلى :

$$S^2_{|\lambda_1|} = .419 \qquad (36)$$

$$S^2_{|\lambda_2|} = 5.919 \qquad (41)$$

وبمستوى ثقة قدره ٩٥ ٪ نجد أن القيمة المطلقة للجذر λ_1 لا يمكن أن تزيد عن ٢ر١١٠ أو تقل عن — ٠ر٤٢٦ كما أن القيمة المطلقة للجذر λ_2 لا يمكن أن تزيد عن ٧ر٠١٦ أو تقل عن ٢ر٥٢١٠ . ويتضح من ذلك أن القيمة المطلقة سواء للجذر λ_1 أو λ_2 لا تختلف جوهريا عن الواحد الصحيح وذلك بمستوى معنوية ٥ ٪ . ونستخلص من ذلك أنه ليس من المحتمل أن يكون نموذج الاقتصاد المصرى مستقرا .

٤ — التحليل الديناميكى للمضاعف :

وختاما نتعرض لأثر التغير بوحدة نقدية واحدة فى كل من العرض النقدى M_t والانفاق الحكومى G_t على المتغيرات الداخلية ، ويتطلب ذلك حساب مضاعفات الأجل القصير والأجل المتوسط والأجل الطويل .

وتمثل عناصر المصفوفة E [أنظر المعادلة (١٥)] مضاعفات الأجل القصير impact multipliers . وهى تقيس الأثر الفورى للتغير بوحدة نقدية واحدة فى أحد المتغيرات الخارجية على أحد المتغيرات الداخلية على فرض أن المتغيرات الاخرى ثابتة ولم تتغير . وتظهر هذه المضاعفات بالجدول رقم (I) .

IS THE EGYPTIAN ECONOMY DYNAMICALLY STABIE ?

A four-equation short—term dynamic model is constructed for the Egyptain economy for the period 1960-1975. Estimates of structural equations are obtained with the aid of the three stage least squares method, and the reduced form is derived . Both deterministic and stochastic aspects of stability issue are examined which could be of importance to Egyptián policy-makers. The equillium time path is also examined of the response of an endogenous variable to a unit shock in one single period (which is not sustained) of an exogenous variable.

1. A Short Term Model For Egypt :

The model to be presented has not had the resources devoted to it that would be necessary to make it a substantive contribution to knowldge about the Egyptian economy. The number of equations and variables has intentially been kept rather small. It was decided to include one consumption equation, one equation to explain investment behavior, one equation for the money market, and the known identity of national income.

All flows of income and output and expenditure are in real terms, no attempt being made here to explain variations in either absolute or relative prices. A major defect in the model is that the supply side is suppressed, neither production function nor labor equation are included, so that variations in output are explained in the crude fashion adopted by many followers of Keynes, namely in terms of effective demand only ignoring resource limitations.

The following notations are used. For the endogenous variables considered in the model we have[1]:

C_t = real private consumption expenditure in period t,

I_t = real gross fixed capital formation in period t,

R_t = real interest rate (nominal interest rate minus actual inflation rate lagged one period),

Y_t = real national income.

For the exogenous variables, we have :

$(X-M)_t$ = real net exports in period t,

N_t = population size in millions in period t,

M_t = real money supply (currency plus demand deposits, i.e, M_1) in period t,

G_t = real sum of goverment expenditure, inventories and statistical corrections.[2]

The dynamic model is :

$$C_t = a_1Y_t + a_2C_{t-1} + a_3N_t + a_4 + U_{ct} \tag{1}$$

$$I_t = b_1R_t + b_2Y_t + b_3I_{t-1} + b_4(X-M)_t + b_5N_t + b_6 + U_{it} \tag{2}$$

$$R_t = c_1Y_t + c_2M_t + c_3 + U_{rt} \tag{3}$$

$$Y_t = C_t + I_t + (X-M)_t + G_t \tag{4}$$

As indicated by equation (1) private consumption C_t is assumed to depend on the income, the lagged private consumption, and the population size. This equation takes into account the adjustment of actual to desired consumption, the

the latter depending on current real income and on the population size.

The investment equation (2) considers the adjustment of actual to desired investment, the latter depending on the current real interest rate, the real income, the real net exports, and the population size.

The consumption equation (1) with the investment equation (2) plus the accounting identity of national income (4) form the so-called IS-equation. The other split of the keynesian model known as LM-equation is given by equation (3) which is a somewhat unusual keynesian money demand function. Assuming a unit-income-elasticity of the demand for money - this is probably the case in Egypt-taking the linear approximation of the logarithmic transformation of the original function, and putting the real interest rate variable on the left hand side, one arrives at the given equation (4).

For our investigations we use annual data from 1960 to 1975 as published in International Financial Statistics. The data appeared in these publications are nominal, while the data used in running the regression are real. They are deflated by converting them to 1970 prices. The flows are measured in millions of 1970 Egyptian pounds per year except for the interest rate, which is measured in percentage points. The data appear in the appendix.

by examining the order and rank conditions for identifiability of each equation, it turns out that the whole model is overidentified. This excludes indirect least squares as a procedure for estimation. The three stage least squares method (Intriligator, 1978, PP. 402-412) has been used to estimate the structural equations, and the results are as follows (numbers in parentheses are standard errors of estimates)[5]:

$$C_t = \underset{(.151)}{.738}\, Y_t - \underset{(.249)}{.694}\, C_{t-1} + \underset{(13.631)}{32.623}\, N_t + \underset{(145.039)}{51.020} + U_{ct} \quad (5)$$

$$I_t = -\underset{(189.874)}{301.438}\, R_t + \underset{(.491)}{.385}\, Y_t + \underset{(1.298)}{.980}\, I_{t-1} + \underset{(.221)}{.084}(X-M)_t - \underset{(53.613)}{17.021}\, N_t + \underset{(656.978)}{986.020} + U_{it} \quad (6)$$

$$R_t = \underset{(.0006)}{.0016}\, Y_t - \underset{(.0012)}{.0023}\, M_t + \underset{(.9335)}{2.490} + U_{rt} \quad (7)$$

$$Y_t = C_t + I_t + (X - M)_t + G_t \quad (8)$$

Our structral economic model can be rewritten as :

$$AZ_t = EZ_{t-1} + CX_t + U_t$$

Where :

$$Z_t = \begin{bmatrix} C \\ I \\ R \\ Y \end{bmatrix}_t, Z_{t-1} = \begin{bmatrix} C \\ I \\ R \\ Y \end{bmatrix}_{t-1}, X_t = \begin{bmatrix} (X-M) \\ N \\ M \\ G \end{bmatrix}_t, \text{ and } U_t = \begin{bmatrix} U_c \\ U_i \\ U_r \\ 0 \end{bmatrix}_t$$

Because the system contains 4 equations, each of the matrices A and B has dimensions 4 x 4, while the matrix C has dimensions 4 x 5. The elements of the matrices A,B, and C are constants, many of which are assumed to be zero. The zeroes indicate, of course, that certain variables do not appear in a particular equation. The estimates of A,B, and C are given below :

$$A = \begin{bmatrix} 1 & 0 & 0 & -.738 \\ 0 & 1 & 301.438 & -.385 \\ 0 & 0 & 1 & -.0016 \\ -1 & -1 & 0 & 1 \end{bmatrix} \qquad (10)$$

$$B = \begin{bmatrix} -.694 & 0 & 0 & 0 \\ 0 & .980 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \qquad (11)$$

$$C = \begin{bmatrix} 0 & 32.623 & 0 & 0 & 51.060 \\ .084 & -17.021 & 0 & 0 & 986.020 \\ 0 & 0 & -.0023 & 0 & 2.490 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad (12)$$

2. The Reduced Form and Stability :

To investigate the questions of stability we need to write the system in "reduced form", i.e., by solving with respect to the endogenous variables.[6] Given the linearity of the equations, and given also the fact that the lags are confined to one year, we can write the reduced form as follows:

$$Z_t = DZ_{t-1} + EX_t + V_t \tag{13}$$

where

$$D = A^{-1}B, \quad E = A^{-1}C, \quad \text{and} \quad V_t = A^{-1}U_t$$

Thus, the system (13) describes each of the endogenous variables in year t linearly in terms of the same variables lagged one year (Z_{t-1}), of the exogenous variable in the same year (X_t), and of the reduced form disturbances (V_t) which are linear combinations of the structural disturbances (U_t). The reduced form coefficient matrices, D and E are of order 4x4 and 4 x 5, respectively. Their numerical values, corresponding with the estimated coefficients in (10)-(12) are given below:

$$D = \begin{bmatrix} -2.119 & 2.013 & 0 & 0 \\ .166 & .715 & 0 & 0 \\ -.003 & .004 & 0 & 0 \\ -1.932 & 2.728 & 0 & 0 \end{bmatrix} \tag{14}$$

$$E = \begin{bmatrix} 2.227 & -64.671 & 1.424 & 2.054 & 639.527 \\ -.210 & -21.246 & .506 & -.271 & 157.853 \\ .005 & .069 & .001 & .004 & 3.766 \\ 3.017 & 43.423 & 1.930 & 2.783 & 797.380 \end{bmatrix} \tag{15}$$

In extenso, the reduced form system is :

$$C_t = -2.119\, C_{t-1} + 2.013\, I_{t-1} + 2.227(X-M)_t + 64.671\, N_t + 1.424\, M_t + 2.054\, G_t + 639.527 + V_{ct} \tag{16}$$

$$I_t = .188\, C_{t-1} + .715\, I_{t-1} - .210\, (X-M)_t - 21.246\, N_t + .506\, M_t - .271\, G_t + 157.853 + V_{it} \tag{17}$$

$$R_t = -.003\, C_{t-1} + .004\, I_{t-1} + .005\, (X-M)_t + .069\, N_t + .001\, M_t + .004\, G_t + 3.766 + V_{rt} \tag{18}$$

$$Y_t = -1.932\, C_{t-1} + 2.728\, I_{t-1} + 3.017(X-M)_t + 43.423\, N_t + 1.930\, M_t + 2.783\, G_t + 797.380 + V_{yt} \tag{19}$$

Lagging this reduced form (13) one period and substituting back in we have :

$$Z_t = D\,(\,DZ_{t-2} + EX_{t-1} + V_{t-1}) + EX_t + V_t \tag{20}$$

Applying this procedure m times yields :

$$Z_t = D^{m+1}\, Z_{t-m-1} + \sum_{i=0}^{m} D^i\, EX_{t-i} + \sum_{i=0}^{m} D^i\, V_{t-i} \tag{21}$$

To investigate the inherent dynamic properties of our model, it is convenient to take m=t-1 in (21)[7] ;

$$Z_t = D^t Z_o + \sum_{i=o}^{t-1} D^i EX_{t-i} + \sum_{i=o}^{t-1} D^i\, V_{t-i} \tag{22}$$

This equation expresses the time path of the endogenous variables in terms of three components : (initial conditions (Z_o) the time path of the exogenous variabes (X_{t-i} for i = 0,..., t-1). The inherent dynamic properties refer to the characteristics of the time path undertaken by the endogenons variables following an initial displacement from equilibrium without further exogenous changes or disturbances. Therefore, let us take both $X_{t-i} = 0$ and $V_{t-i} = 0$ for all $i \geq 0$ in (22); this leaves

$$Z_t = D^t Z_o \tag{23}$$

A well-known theorem (Zurmühl, 1958, P. 181) states that any square matrix such as D, which has no multiple non-zero characteristic roots, can be written as the sum of as many matrices of rank one as the rank of this square matrix amounts to, in such a way that each of these separate matrices of rank one is equal to the product of a non-zero characteristic root of that matrix and the corresponding characteristic column and row.[8] Thus, if λ_i is the i^{th} root of the matrix D, R_i is the product of the corresponding characteristic column and row, and N is the rank of D; we can write :

$$D = \sum_{i=1}^{N} \lambda_i \; R_i \tag{24}$$

Furthermore, if we raise D to the t^{th} power, the roots become λ_i^t. While the characteristic vectors, and hence their product, remain unchanged. Thus:

$$D^t = \sum_{i=1}^{N} \lambda_i^t \; R_i \tag{25}$$

Inserting (25) into (23) gives:[9]

$$Z_t = \sum_{i=1}^{N} \lambda_i^t \; R_i \, Z_o \tag{26}$$

which shows that the inherent dynamic properties of this model of the Egyptian economy are intimately connected w[illegible] the characteristic roots of the matrix D. It is clear th[illegible] that the stability condition.

$$\lim_{t \to \infty} D^t = \lim_{t \to \infty} \sum_{t=1}^{N} \lambda_i^t \quad R_i = 0 \qquad . \qquad (27$$

requires that each of the λ_i's be less than one in absolu[illegible] value. Now, if all λ_i's are positive (and less than one): the time path of endogenous variable Z_t is monotonically decreasing to zero and if some λ_i's are negative (and less than one), we may have an oscillating Z_t but eventually approaching zero (Baumol, 1959, PP. 197-201).

For the D-matrix of our model as specified by (14), th[illegible] characteristic equation becomes :

$$\begin{vmatrix} -2.119-\lambda & 2.013 & 0 & 0 \\ .188 & .715-\lambda & 0 & 0 \\ -.003 & .004 & -\lambda & 0 \\ -1.932 & 2.728 & 0 & -\lambda \end{vmatrix} = 0 \qquad (28)$$

or equivalently

$$\begin{vmatrix} -2.119-\lambda & 2.013 \\ .188 & .715-\lambda \end{vmatrix} = 0 \qquad (29)$$

This last equation can be reduced to the following characteristic polynomial :[10]

$$f(\lambda) = \lambda^2 + 1.405\lambda - 1.893 = 0 \qquad (30)$$

Which has the following two real roots :

$$\lambda_1 = .842$$

$$\lambda_2 = -2.247$$

As we can see, the second root is greater than unity in absolute value, and thus model is unstable and shows explosive oscillations.

3. Stochastic Aspects of Stability Analysis :

The estimates of characteristic roots are sample statistics they are after all ultimately derived from the estimates of the structural parameters - and hence subject to sampling error. Following Theil and Boot (1962), the asymptotic variance of the absolute value of λ_i is given by:

$$s^2(\lambda_i) = k_i' \; W_D \; k_i \tag{31}$$

Where k_i' denotes the row vector of successive rows of the matrix H_i of cofactors of $D - \lambda_i I$ divided by trace H_i, the sum of the diagonal elements. W_D is the variance-covariance matrix of the elements of D, a submatrix of W. Goldberger, Nagar, and Odeh (1961) showed that the variance-covariance matrix W of derived reduced form coefficients in (14) and (15) is given by :

$$W = G \; Q \; G' \tag{32}$$

G is a submatrix of $D^{-1} \otimes (F \; \vdots \; I)$ with those columns deleted which correspond to zero or unit restricted structural parameters, where $F = (D \; \vdots \; E)$ and $\otimes$ is the Kronecker product.[11]

W is the variance-covariance matrix of the structural coefficients in (10)-(12). The matrices Q, G, and W_D are given in the appendix.

To test absolute value of λ_1 we construct the matrix

$$D - \lambda_1 I = \begin{bmatrix} -2.962 & 2.013 \\ .188 & -.128 \end{bmatrix} \tag{33}$$

The matrix of the cofactors of this matrix is :

$$H_1 = \begin{bmatrix} -.128 & -.188 \\ -2.013 & -2.962 \end{bmatrix} \tag{34}$$

Noting that the sum of the diagonal elements of this matrix is -3.090 we construct the following column vector

$$k_1 = \begin{bmatrix} .041 \\ .061 \\ .652 \\ .959 \end{bmatrix} \tag{35}$$

Hence, the estimated asymptotic variance of $|\lambda_1|$ is :

$$s^2_{|\lambda_1|} = .419 \tag{36}$$

This corresponds to a standard error of .647. We compute the following 95% confidence interval:[12]

$$p\ (-.426 \leq |\lambda_1| \leq 2.110\ ) = .95 \tag{37}$$

Hence, we conclude that the absolute value of λ_1 is not significantly different from unity at the 5% significance level.

To test the modulus of the second root which is $|\lambda_2| = 2.247$, we have :

$$D - \lambda_2 I = \begin{bmatrix} .128 & 2.013 \\ .188 & 2.962 \end{bmatrix} \qquad (38)$$

$$H_2 = \begin{bmatrix} 2.962 & -.188 \\ -2.013 & .128 \end{bmatrix} \qquad (39)$$

$$k_2 = \begin{bmatrix} .959 \\ -.061 \\ -.651 \\ .041 \end{bmatrix} \qquad (40)$$

The variance of the absolute value of our second root is

$$s^2_{|\lambda_2|} = 5.919$$

Which corresponds to a standard error of 2.433, and the 95% confidence interval is:

$$p\ (-2.521 \leq |\lambda_2| \leq 7.016) = .95 \qquad (41)$$

Again, instability can not be ruled out. Hence, we can say that it seems unlikely that the Egyptian economy is stable.

4. Dynamic Multiplier Analysis

We are finally interested in the effects of the exogenous variables M_t (real money supply) and G_t (real government expenditures) on the endogenous variables.

The impact multipliers are elements of the matrix E, as stated in (15), and measure the immediate effect of a one unit change in an exogenous variable on an endogenous variable keeping all other variables constant:

	M_t	G_t
C_t	1.424	2.054
I_t	.506	-.271
R_t	.001	.004
Y_t	1.930	2.783

All the multipliers have the expected sign except the one from M_t on R_t.

The interim or delayed multipliers are elements of D^iE, and give the effect on the current value of an endogenous variable of a one unit change in an exogenous variable after a period of i years.[13] Based on this equation we present the estimated delay effects of an exagenous change of one million Egyptian pounds in the money stock and goverment expenditures for lags up to 5 years:[14]

A. Delayed multipliers of change in M_t :

Year	C_t	I_t	R_t	Y_t
0	1.424	.506	.001	1.930
I	-2.000	.629	-.002	-1.371
2	5.506	.074	-.313	5.579
3	-11.522	5.022	.247	-10.434
4	26.608	3.506	-.330	25.219
5	-59.190	4.588	.932	-55.180

B. Delay multipliers of change in G_t :

Year	C_t	I_t	R_t	Y_t
0	2.054	-.271	.005	2.783
1	-4.899	.192	-.008	-4.706
2	10.770	-.783	.016	9.986
3	-24.403	-.643	.344	-22.937
4	54.670	-2.042	-.826	51.127
5	-122.993	1.651	1.811	-115.245

Summarizing the paper we might say that an application of the concept of equilibrium and stability as well as some stochastic considerations to a simple dynamic model of the Egyptian economy has indicated certain aspects of economic stability which could be of importance to policy-makers.

Appendix

In this appendix we give the data series used in this study, and the variance-covariance matrix Q of the structural coefficients. Furthermore, we present the transformation matrix G, and the variance-covariance matrix W_D of the reduced form lagged-endogenous-coefficients.

1. Data series used in estimation and testing (in millions of Egyptian pounds):

Year	C_t	I_t	Y_t	R_t	C_{t-1}	I_{t-1}	N_t	$(X-M)_t$	M_t	G_t
1960	1397	245	1978	3.303	1353	218	25.92	3	580.8	327
1961	1419	322	2081	3.298	1397	245	26.58	-27	648.7	365
1962	1617	369	2222	5.319	1419	322	27.26	-126	649.9	363
1963	1707	437	2448	5.314	1617	369	27.95	-150	751.7	464
1964	1754	532	2646	5.289	1707	737	28.66	-188	873.3	565
1965	1791	438	2692	5.183	1754	532	29.39	-70	801.1	535
1966	1779	424	2675	5.110	1791	438	30.14	-138	768.4	542
1967	1821	400	2739	5.103	1779	424	30.91	-27	768.4	544
1968	1892	313	2679	5.068	1821	400	31.69	-38	774.4	603
1969	1876	346	2754	5.037	1892	313	32.50	-69	774.7	670
1970	1940	350	2927	5.000	1876	346	33.33	-121	782.8	717
1971	2004	344	2993	4.969	1940	350	34.08	-153	820.8	770
1972	2124	385	3232	4.947	2004	344	34.84	-182	939.5	859
1973	2130	421	3310	4.902	2124	385	35.62	-180	1097.3	929
1974	2127	526	3245	4.783	2130	421	36.42	-378	1234.8	901
1975	2467	920	3530	4.665	2127	526	37.23	840	1395.2	909

11- Variance-covariance Matrix Q of the structural coefficients (13x13):

	a_1	a_2	a_3	a_4
a_1	.023			
a_2	-.027	.062		
a_3	-.090	-1.050	185.798	
a_4	12.782	-5.486	-1474.747	21036.346
b_1	-2.683	4.470	21.503	-1297.501
b_2	.004	.000	-.357	.942
b_3	.014	-.021	-.232	7.392
b_4	-.001	.001	.052	-.805
b_5	-.346	-.033	45.573	-416.927
b_6	6.404	-12.629	-459.048	14059.927
c_1	-.000	.000	.000	-.015
c_2	.000	-.000	-.000	.014
c_3	.000	-.015	-.117	31.468

	b_1	b_2	b_3	b_4	b_5	b_6
b_1	36052.156					
b_2	25.040	.241				
b_3	-214.277	-.393	1.685			
b_4	35.207	.032	-.228	.049		
b_5	-3117.398	-25.753	43.405	-3.975	2874.367	
b_6	-61840.923	174.826	102.100	-44.254	-20870.137	431620.431
c_1	.057	.000	-.000	.000	-.004	-.159
c_2	-.131	-.000	.001	-.000	.009	.254
c_3	-45.912	-.076	.279	-.035	3.300	223.371

	c_1	c_2	c_3
c_1	.000		
c_2	-.000	-.000	
c_3	-.000	.000	.871

iii. Transformation matrix G (24x13) :

a_1	a_2	a_3	a_4
5.699	3.054	0	0
6.330	0	0	0
9.214	0	0	0
132.614	0	3.054	0
5.893	0	0	0
6.500	0	0	0
2435.191	0	0	3.054
.523	-.271	0	0
-.739	0	0	0
-.817	0	0	0
-11.759	0	-.271	0
-.523	0	0	0
-.754	0	0	0
-215.935	0	0	-.271
-.009	-004	0	0
.012	0	0	0
.013	0	0	0
.193	0	.004	0
.009	0	0	0
.012	0	0	0
3.551	0	0	.004
-5.376	2.783	0	0
7.591	0	0	0
8.397	0	0	0
120.855	0	2.783	0
5.370	0	0	0
7.746	0	0	0
2219.256	0	0	2.783

b_1	b_2	b_3	b_4	b_5	b_6
-.006	-3.967 *	0	0	0	0
.009	5.602	2.054	0	0	0
.010	6.197	0	2.054	0	0
.14[illegible]	89.191	0	0	2.054	0
.002	3.963	0	0	0	0
.009	5.717	0	0	0	0
7.735	1637.811	0	0	0	2.054
-.002	-1.408	0	0	0	0
.003	1.989	.729	0	0	0
.004	2.200	0	.729	0	0
.051	31.564	0	0	.729	0
.001	1.407	0	0	0	0
.003	2.029	0	0	0	0
2.746	581.445	0	0	0	.729
-.000	-.009	0	0	0	0
.000	.012	.004	0	0	0
.000	.013	0	.004	0	0
.000	.193	0	0	.004	0
.000	.009	0	0	0	0
.000	.012	0	0	0	0
.017	3.551	0	0	0	.004
-.009	-5.376	0	0	0	0
.012	7.591	2.783	0	0	0
.013	8.397	0	2.783	0	0
.193	120.855	0	0	2.783	0
.002	5.370	0	0	0	0
.012	7.746	0	0	0	0
10.4[illegible]	[illegible]19.256	0	0	0	2.783

1195.907	0	0
-1688.745	0	0
-1867.959	0	0
-26885.509	0	0
-1194.714	-619.150	0
-1723.209	0	0
-493698.353	0	-619.150
424.563	0	0
-599.527	0	0
-663.151	0	0
-9544.720	0	0
-424.140	-219.807	0
-611.763	0	..
-175269.605	0	-219.807
.661	0	0
-.934	0	0
-1.03	0	0
-14.865	0	0
-.661	-.342	0
-.953	0	0
-272.968	0	-.342
1620.470	0	0
-2288.272	0	0
-2531.109	0	0
-36430.229	0	0
-1618.853	-838.957	0
-2334.972	C	0
-666967.959	0	-838.957

iv. Variance - covariance matrix W_D of the reduced form l
endogenous - coefficients:

8.037			
-5.039	6.094		
1.806	-.841	.779	
-.963	1.315	-.359	.478

التعليمات

Footnotes.

[1] Endogenous variables are variables whose values are determined by the model, and differ from exogenous variables whose values are determined outside the model and considered as given inputs to the model.

[2] M_t and G_t may be viewed as policy variables that can be manipulated at will by the policy-makers.

[3] Dynamic models are models whose predetermined variables include lagged endogenous variables. Dynamic models do not only specify how predetermined variables at a point in time generate the dependent variables at that point, but also specify how the time path of the exogenous variables generates the time path of the endogenous variables over a span of time (Goldberger, 1964, P. 373).

[4] There are two means of adjustment of the money market to disequilibrium between supply and demand. The adjustment could take place through income. The obvious alternative is the rate of interest which can act as the market clearing mechanism like other "prices". In this case therefore it is the rate of interest which is the dependent variable and not the money stock (Mayes, 1981, P. 186).

[5] The implied elasticities of the demand for money with respect to income is $\frac{.0016}{.0023} = .7$, and with respect to interest rate is $-\frac{1}{.0023} = -435$ (Mayes, 1981, P. 186).

[6] This requires that the matrix A in (9), which is of order 4x4, be nonsingular.

[7] The power series in (21) or (22), called a Neumann expansion, converges if $\lim_{t \to \infty} D^t = 0$ or, equivalently, if all characteristic roots of D have modulus less than unity (Nikaido, 1972). That is, all characteristic roots of D must lie inside the unit circle in the complex plane, and this provides a stability condition for the model as we will see soon (Glaister, 1972, ch.9).

[8] The characteristic roots of the matrix D are the solutions of the characteristic equation:

$$|D - \lambda I| = 0$$

[9] In the simplest one dimensional case, i.e., N = 1 we have

$$Z_t = \lambda^t Z_o$$

Tintner, Bohm, and Rieder (1979, P. 67) used this relation to develop a new concept of equilibrium and stability by concentrating on the time t it takes until a disturbance is reduced to half its original value.

[10] The characteristic polynomial of a matrix D of order nxn is given by the following

$$\lambda^n - (trD)\,\lambda^{n-1} + (tr_2 D)\,\lambda^{n-2} - (tr_3 D)\,\lambda^{n-3} + \dots -(-1)^n (tr_{n-1} D)\,\lambda + (-1)^n\,|D| = 0$$

Where tr_k is defined to be the sum of the determinants of $\binom{p}{k}$ matrices of order kxk which can be formed by intersecti any k rows of D with the same k columns (Beyer, 1979, P.

[11] The Kronecker product of a matrix with a matrix in olves multiplying each element of the matrix on the left b the entire matrix on the right (Intriligator, 1978, P. 578

[12] This is computed as follows :

$$|\lambda_1| \pm 1.96 \quad s_{|\lambda_i|}$$

[13] The total multipliers which are given by $(I-D)^{-1}E$ can not be computed because the convergence condition is n met (Theil and Boot, 1962, P. 621). The total multipliers the limit, if they exist, of the corresponding intermediate run multipliers as t approaches infinity, which are given b the following matrix.

$$(I+D+D^2+\ldots+D^{t-1})\,E = (I-D)^{-1}\,(I-D^t)\,E$$

the limit of this expression exists as t approaches infinit if and only if the limit of D^t exists, which is the case if the characteristic roots of D are all smaller than one in absolute value. This last condition is necessary and suffi ient for the time path of the endogenous variables to reach a steady state. Of course, if the time path never reaches a steady state in the first place, how can we speak of the derivative of the steady state with respect to permanent values of the exogenous variables?

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