AGGREGATION AND STABILITY OF U.S. IMPORT DEMAND FUNCTION

This study is concerned with the analysis of the common practice of using highly aggregated data when testing for the presence of structural change in the parameter vector underlying U.S. import demand function. Only the cases for aggregation over commodities and aggregation over time periods are considered. Moreover, structural change is assumed to be detected using the so-called CUSUM-SQ-test, which is based upon CUmulated SUMs of SQuared recursive residuals. The analysis shows that aggregation over commodities may result in stable macro parameter vectors where the underlying micro vectors are unstable or vice versa. It also shows that aggregation over time periods, say from monthly to quarterly, tends in general to stabilize the estimates of parameter vectors over the investigation period. Supporting evidence for these findings was found using U.S. monthly imports of cocoa, natural rubber, and unmanufactured wool for the period from July 1959 to June 1979, and with estimation and testing processes carried out at two levels for each type of aggregation.

The international trade literature includes studies of the stabilit of the aggregate import demand function of the United States. The most recent, that of Stern et al. (1979), utilizes an analysis of the recursive residuals to test for stability. That study, however, does not consider the effect that either temporal or commodity aggregation may have on results. There are also some studies (e.g., Rhomberg and Boissonneault, 1965) that consider subaggregates as well as aggregate commodities. Specific inferences about commodity aggregation are very infrequent. The present study, by examining both commodity and temporal aggregation problems has attempted to rectify these omissions.

Our study proceeds as follows. In Section I, we describe the CUSUM-SQ test. Section II analyzes the influence of aggregation on the CUSUM-SQ test statistics. The estimation model is outlined in Section III. We then report in Section IV the empirical results. Some concluding remarks are given in Section V.

I. The CUSUM-SQ Test

The CUSUM-SQ test is suggested by Brown et al. (1975) to monitor the constancy of a regression relation in time. Formally speaking, it is designed to test the following null hypothesis of no structural change:

$$H_0: B_t = B \mid \sigma_t^2 = \sigma^2 \quad \forall\, t \in T \tag{1}$$

For H_0, there is a class of alternatives (Wilson, 1978, p. 68). However, for the purpose of this study, it suffices to express the alternative hypothesis in its general form, namely

$$H_A: \text{Not } H_0. \tag{2}$$

The CUSUM-SQ test is based on recursive residuals which are defined to be proportional to one-period forecast errors obtained from regressions performed over successively longer sample ranges.[1] Consider the following model:

$${}_TY_{t_1} = {}_TX_{t_K}\,B_{t_1} + {}_TU_{t_1}, \qquad t=1,2,\ldots,T \tag{3}$$

where $Y_t=(y_1\ y_2\ \ldots\ y_T)'$ is a column vector of T ordered observations on the endogenous variable Y, $X_t=(x_1\ x_2\ \ldots\ x_T)'$ is a matrix of known non-stochastic explanatory variables such that $(X_t'X_t)$ has full rank K ($<T$), and $B_t=(b_1\ b_2\ \ldots\ b_K)'$ is a column vector of unknown coefficients. b_1 is the intercept if the first explanatory variable is set identically equal to one. The Stochastic vector $U_t=(u_1\ u_2\ \ldots u_T)'$ is assumed to be drawn from a Gaussian distribution with the characteristics that the

disturbances are serially uncorrelated zero mean homoscedastic random variables.[2] The index t characterizes the time dependence of its constituents. Let $\hat{B}_{r-1}$ be an OLS estimate of B based on r-1 observations. The forecast errors, v_r, for periods beginning at r=K+1, K+2, ..., T are

$$v_r = y_r - x_r\hat{B}_{r-1}, \qquad r=K+1,\ K+2,\ \ldots,\ T \tag{4}$$

and the series of (T-K) recursive residuals, w_r, is given by

$$w_r = v_r \,/\, (d_{r-1})^{1/2}, \qquad r=K+1,\ K+2,\ \ldots,\ T \tag{5}$$

where

$$d_{r-1} = 1 + x_r(X'_{r-1}X_{r-1})^{-1}x'_r \tag{6}$$

X_{r-1} contains only the first r-1 rows of X_t of (3). d_{r-1} is the proportionality factor. It has been introduced in order to let w_r have the same scalar variance-covariance matrix that the original residuals of (3) have.[3] Then, the CUSUM-SQ series, C_r, is defined to be

$$C_r = \frac{\sum_{i=K+1}^{r} w_i^2}{\sum_{i=K+1}^{T} w_i^2},^{4} \qquad r=K+1,\ K+2,\ \ldots,\ T \tag{7}$$

The denominator in (7) is just a constant scaling factor that makes $0 \leq C_r \leq 1$. From lemma 2 in Brown et al. (1975, p. 152)

$$S_r = S_{r-1} + w_r^2 \tag{8}$$

where S_r denotes the squared sums of residuals obtained from running a regression on the first r observations only. S_{r-1} is defined similarly. It follows from (8) that

$$S_r = \sum_{i=K+1}^{r} w_i^2, \qquad r=K+1,\ K+2,\ \ldots,\ T \tag{8a}$$

and

$$S_T = \sum_{i=K+1}^{T} w_i^2. \tag{8b}$$

The substitution of (8a) and (8b) into (7) results in

$$C_r = S_r \,/\, S_T, \qquad r=K+1,\ K+2,\ \ldots,\ T \tag{7a}$$

The statistic C_r is thus the proportion of the total sum of squared residuals (obtained by using all T observations) that has been cumulated through the $(r-K)^{th}$ regression. Expressing the CUSUM-SQ series in terms of OLS residuals rather than recursive residuals has the advantage of reducing the algebraic effort to be done in exploring the effects of aggregation on C_r. Thus, in the next section (7a) not (7) will be used.

Under the null hypothesis H_0, C_r has the Beta distribution with $E(C_r)=(r-K)/(T-K)$, which takes value zero when $r<K+1$ and one when $r=T$, i.e., $0 \leq E(C_r) \leq 1$.

To perform the CUSUM-SQ test, the following test statistic is, then, calculated:

$$C_c = \max \left| C_{K+2z} - \frac{z}{Z} \right| = \max\ (c^+,\ c^-),\ z=1,\ 2,\ \ldots,\ Z-1 \tag{9}$$

where $Z=(T-K)/2$, and c^+ and c^- are the maximum positive and negative deviations, respectively, of the set of Z statistics (C_{K+2}, C_{K+4}, ..., C_{T-2}) from their respective mean values. The tabulated critical values, C_t, are provided by Durbin (1969, p. 4). To find C_t, enter the table of critical values at $n=(1/2)(T-K-2)$ and take the value under $1/2\ \alpha$ if the test is two-sided. The hypothesis of structural stability is then rejected if $C_c > C_t$.[5]

In addition, confidence belts can be constructed as follows:[6]

(1) Plot C_r against time.

(2) Draw the expected mean line by joining the points (K, 0) and (T, 1).

(3) For a two-sided test, draw a pair of lines $\pm C_{t(\alpha)} + (r-K)/(T-K)$ parallel to the expected mean line, where α is the desired level of significance. H_0 is rejected if the sample path crosses either line.

II. Theoretical Analysis

It is the intention of this section to analyze the consequences of the common practice of employing aggregated data when testing functional relationships in international trade for structural change. The case of aggregation over commodities is treated first, and it is followed by the case of aggregation over time periods. The temporal aggregation is considered in a finite distributed lag framework which includes simple models as special case.

Commodity Aggregation

Starting with the following micro model for the i^{th} commodity in period jl:

$$ {}_{mQ}Y_{ijl_1} = {}_{mQ}X_{ijl_K}B_{ijl_1} + {}_{mQ}U_{ijl_1}, \tag{10} $$

i=1,2,...,n (commodities)

j=1,2,...,m-1 (micro time units)

l=1,2,...,Q (macro time units).

where Y, X, B, and U are defined as in (3). Furthermore, let

$$Y_{jl} = \sum_{i=1}^{n} Y_{ijl},\ X_{jl} = \sum_{i=1}^{n} X_{ijl},\ \text{and } U_{jl} = \sum_{i=1}^{n} U_{ijl} \tag{11}$$

then the macro model is:

$$_{mQ}Y_{jl_1} = {}_{mQ}X_{jl_K}B_{jl_1} + {}_{mQ}E_{jl_1}, \tag{12}$$

j=1,2,...,m-1

l=1,2,...,Q.

For this macro model to be consistent with the micro model (10) one of the following alternative conditions must hold:

(a) $E_{jl} = U_{jl} + \sum_{i=1}^{n} X_{ijl}\,(B_{ijl} - B_{jl})$;

(b) $E_{jl} = U_{jl}$ and $B_{sjl} = (\sum_{i=1}^{n} X_{sijl}\, B_{sijl})/X_{sjl}$, s=1,2,...,K

where B_{sjl}, B_{sijl}, X_{sjl}, and X_{sijl} are the s[th] elements of B_{jl}, B_{ijl}, X_{jl}, and X_{ijl} respectively; or

(c) $E_{jl} = U_{jl} + \sum_{i=1}^{n} V_{ijl}\, B_{ijl}$ and $B_{jl} = \sum_{i=1}^{n} W_{ijl}\, B_{ijl}$,

where W_{ijl} and V_{ijl} are the parameter and error term vectors, respectively, of the following auxiliary regression equation (Theil, 1954):

$$_{mQ}X_{ijl_1} = {}_{mQ}X_{jl_K}W_{ijl_K} + {}_{mQ}V_{ijl_1} \tag{13}$$

Anderson (1980, p. 372) has argued that in (a) the error process is correlated with the regressors X_{jl}, and thus has non-zero expectations, while in (b) this problem is surmounted at the expense of having to estimate parameters that depend upon the particular design matrix. If condition (c) is assumed, Wu (1973, p. 787) has claimed that both of the problems are present. It is worthwhile to mention that if the strong assumption of Theil's perfect aggregation, i.e., $B_{ijl} = B_{jl}$ for all i, could be imposed, then, the three alternatives would collapse into only one condition, specifically

$$E_{jl} = U_{jl} = \sum_{i=1}^{n} U_{ijl} \tag{14}$$

In one sense it is not obvious why conditions (a), (b), and (c) present a problem since it is not clear which parameter vector and sum of squared residuals are being estimated. For this reason, and to facilitate the analysis we will assume all micro regressions share a common parameter vector and hence work with (14).

Given the relation in (14), the crux of the problem can be stated in the following inequality:

$$\sum_{l=1}^{Q} \sum_{j=0}^{m-1} (e_{jl})^2 \neq \sum_{l=1}^{Q} \sum_{j=0}^{m-1} \sum_{i=1}^{n} (u_{ijl})^2 \tag{15}$$

where e_{jl} and u_{ijl} are OLS estimates of E_{jl} and U_{ijl}, respectively. This inequality points up the inconsistencies in stability of macro versus micro parameter vectors. This can be shown as follows:

$$\begin{aligned} \sum_{l=1}^{Q}\sum_{j=0}^{m-1}(e_{jl})^2 &= \sum_{l=1}^{Q}\sum_{j=0}^{m-1} \left(\sum_{i=1}^{n} u_{ijl}\right)^2 \\ &= \sum_{l=1}^{Q}\sum_{j=0}^{m-1} \left[\sum_{i=1}^{n}(u_{ijl})^2 \right. \\ &\qquad \left. + 2 \sum_{i=1}^{n-1} \sum_{v=i+1}^{n} u_{ijl}u_{vjl}\right] \end{aligned} \tag{16}$$

which leads to the following relation:

$$S_e = \sum_{i=1}^{n} S_{u_i} + 2 \sum_{i=1}^{n-1} \sum_{v=i+1}^{n} S_{u_i u_v} \tag{17}$$

where $S_e = \sum_{l=1}^{Q} \sum_{j=0}^{m-1} (e_{jl})^2$, $S_{u_i} = \sum_{l=1}^{Q} \sum_{j=0}^{m-1} (u_{ijl})^2$, and

$S_{u_i u_v} = \sum_{l=1}^{Q} \sum_{j=0}^{m-1} u_{ijl} u_{vjl}$. To simplify (17) further; let us assume the

the following:

(1) $S_{u_i} = S_{u_v}$ for all i and v, and

(2) $S_{u_i u_v} = S_{u_h u_f}$ for all i, v, h, and f, $i \neq v$ and $h \neq f$.

Under these assumptions, (17) can be written as:

$$S_e = n\, S_{u_i} + n(n-1)\, S_{u_i u_v} = n\, S_{u_i}\, [1 + (n-1)\, R_u] \tag{18}$$

where

$$R_u = S_{u_i u_v} / (S_{u_i}\; S_{u_v})^{1/2}$$

is the coefficient of intercorrelation among micro residuals.[7]

From the preceding section, macro CUSUM-SQ series, C_r^e, is given by:

$$C_r^e = S_{er} / S_{emQ}; \qquad r=K+1,\ K+2,\ \ldots,\ mQ \tag{19}$$

The substitution of (18) in both the numerator and the denominator of (19) gives:

$$C_r^e = C_r^u \left[\frac{1 + (n-1)\, R_{ur}}{1 + (n-1)\, R_{umQ}} \right]; \quad r=K+1,\ K+2,\ \ldots,\ mQ \tag{20}$$

where C_r^u is the micro CUSUM-SQ series assumed to be common among the n commodities.[8]

From (20), it is clear that if and only if $R_{ur} = R_{umQ}$ for all r, i.e., if R_u is independent of the number of observations, then

$$C_r^e = C_r^u \; ; \qquad r=K+1,\ K+2,\ \ldots,\ mQ \qquad (21)$$

and we should expect unstable micro parameters to yield unstable macro ones, and stable macro parameters to imply stable micro ones. Otherwise, there will be a room for inconsistency in the stability pattern of the micro and macro parameter vectors to exist. This is shown diagramatically in Figure I.A and Figure I.B where it is assumed that R_{ur} does not change sign as r approaches mQ.

In Figure I.A, the intercorrelation coefficients, R_u s, are assumed to possess a positive sign for r=K+1, r=K+2, up to r=mQ. A negative sign is assumed in Figure I.B. The intercorrelation coefficient based on mQ observations, R_{umQ}, is represented by a straight line having a zero slope and an intercept equal to its value. The intercorrelation coefficient based on r observations is illustrated by a curved line starting at the origin and ending equal to R_{umQ}.

Two hypothetical time paths for R_{ur} are given in Figure I.A. In the lower left diagram, the time path for R_{ur} is assumed to lie below the R_{umQ}-line for all r. A different situation is hypothesized in the lower right diagram where at the time r=t* R_{ur} becomes larger than R_{umQ}. The upper diagrams show the implied behavior for the macro CUSUM-SQ series (C_r^e) relative to the micro CUSUM-SQ series (C_r^u). When R_{ur} is below R_{umQ}, absolute deviations from the mean line are larger for C_r^u than for C_r^e. Given the confidence intervals (C.I. line), it is possible for macro parameters to be judged stable where the underly ng micro parameters are, in fact, not stable.

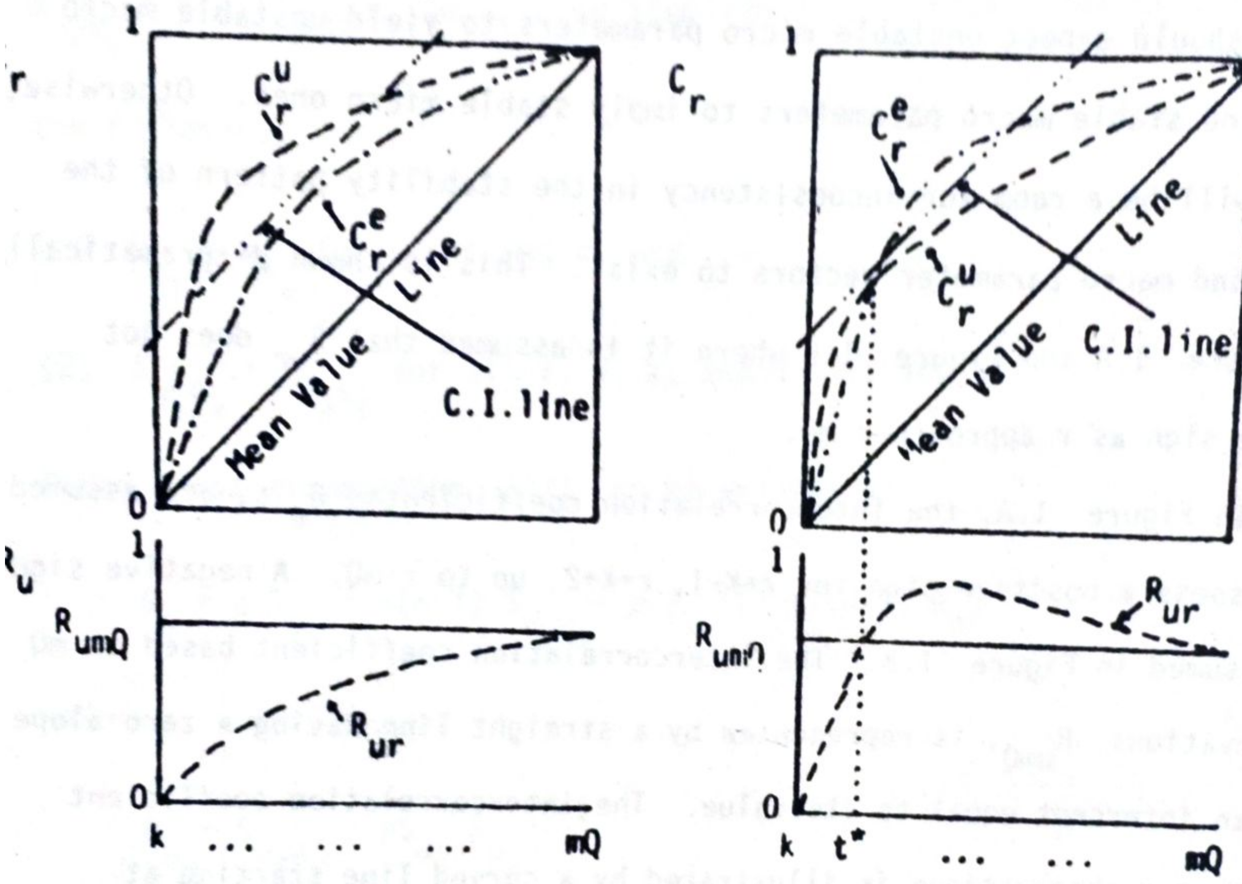

Figure I.A. A hypothetical time path for R_{ur} and implied behavior for C^e_r relative to C^u_r (R_{ur} is assumed to be positive for all values of r).

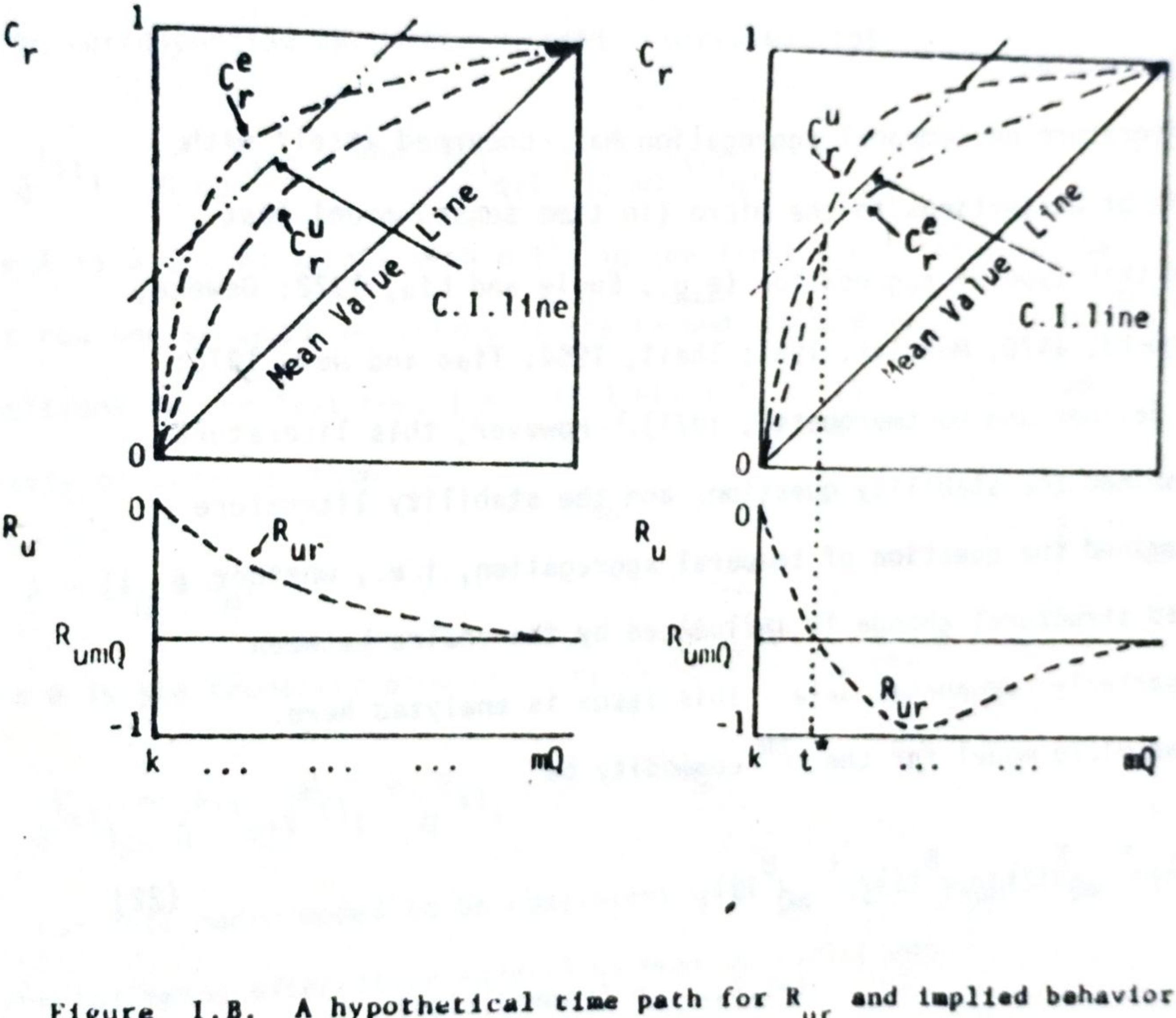

Figure 1.B. A hypothetical time path for R_{ur} and implied behavior for C_r^e relative to C_r^u (R_{ur} is assumed to be negative for all values of r).

In the lower right diagram, as r becomes larger than t* R_{ur} lies above R_{umQ}-line, and an opposite behavior is given in the upper right diagram which indicates that aggregation over commodities may result in unstable macro parameters while the micro parameters are stable. Figure I.B tells a similar story for R_u's less than zero.

Temporal Aggregation

The literature on temporal aggregation has concerned itself with the analysis of distortions to the micro (in time sense) model that result from this type of aggregation (e.g., Engle and Liu, 1972; Geweke, 1978; Moriguchi, 1970; Mundlak, 1961; Theil, 1954; Tiao and Wei, 1975; Wei, 1978; Zellner and Montmarquette, 1971). However, this literature has not examined the stability question, and the stability literature has not examined the question of temporal aggregation, i.e., whether the measured structural change is influenced by the choice between monthly, quarterly, or annual data. This issue is analyzed here.

Let the micro model for the i^{th} commodity be

$$ {}_{mQ}Y_{ijl_1} = {}_{mQ}X_{ijl_{mq+1}} B_{ijl_1} + {}_{mQ}U_{ijl_1} \quad (22) $$

i=1,2,...,n (commodities)

j=0,2,...,m-1 (micro time units)

l=1,2,...,Q (macro time units)

where it is assumed, without loss of generality, that the design matrix X_{ijl} consists of current and mq lagged values of only one explanatory variable, and q is assumed to be less than Q. Each macro time unit Q is assumed to include m micro time units. For example, if Q is a quarter m is three.

The model in (22) can be rearranged to give

$$ {}_{mQ}Y_{ijl_1} = {}_{mQ}Z_{ijl_{q+1}}\delta_{ijl_1} + {}_{mQ}Z^*_{ijl_{(m-1)q}}\delta^*_{ijl_1} + {}_{mQ}U_{ijl_1} \quad (23) $$

where Z_{ijl} includes columns corresponding to the periods 0, m, 2m, ..., mq, Z^*_{ijl} includes the remaining columns of X_{ijl} of (22), and δ_{ijl} and δ^*_{ijl} are the corresponding parameter vectors. Moreover, let

$$ {}_{Q}Y_{il_1} = {}_{Q}A_{mQ}\,Y_{ijl_1}, \text{ and } {}_{Q}X_{il_{q+1}} = {}_{Q}A_{mQ}\,Z_{ijl_{q+1}} \quad (24) $$

where A is a Q x mQ matrix with m 1's in the first m positions in the first row and second m positions in the second row and so on in the Q^{th} m positions in the last row, i.e., if J_m=(1 1 1 ... 1), and I_Q is an identity of order Q, then

$$ A = (I_Q \otimes J_m) \quad (25) $$

where $\otimes$ is the Kronecker product. The macro model is then[9]

$$ {}_{Q}Y_{il_1} = {}_{Q}X_{il_{q+1}}B_{il_1} + {}_{Q}E_{il_1} \quad (26) $$

For this macro model to be consistent with the micro model (22), one of the following alternative conditions must be satisfied:

(a) $E_{il} = AZ_{ijl}(\delta_{ijl}-B_{il}) + AZ^*_{ijl}\delta^*_{ijl} + AU_{ijl}$;

(b) $E_{il} = AZ^*_{ijl}\delta^*_{ijl} + AU_{ijl}$, and $B_{gil} = AZ_{gijl}\delta_{gijl} / X_{gil}$, g=1,2,...,q+1, where B_{gil}, δ_{gijf}, X_{gil}, and Z_{gijl} are the g^{th} elements of B_{il}, δ_{ijl}, X_{il}, and Z_{ijl}, respectively; or

(c) $E_{i1} = (I_Q - X_{i1}(X'_{i1}X_{i1})^{-1}X'_{i1})\, AX_{ij1}B'_{ij1} + AU_{ij1}$, and $B_{i1} = (X'_{i1}X_{i1})^{-1}$ $X_{i1}AX_{ij1}B_{ij1}$ (Theil, 1954, pp. 40-45).

If $\delta_{ij1} = B_{i1}$, i.e., if δ_{ij1} is independent of j which implies that this subset of the micro parameter vector B_{ij1} is stable (constant) within successive m periods, then, the three alternative conditions cited above collapse into[10]

$$E_{i1} = AZ^{*}_{ij1}\delta^{*}_{ij1} + AU_{ij1} \tag{27}$$

In the following discussion, we make use of this assumption and, thus, of relation (27) on the ground that it is not clear which parameter vector and sum of squared errors are being estimated using the temporally aggregated data.

Let the OLS estimate of the macro residual vector be $\hat{E}_{i1}$, i.e.,

$$\hat{E}_{i1} = Y_{i1} - X_{i1}\hat{B}_{i1}$$

where

$$\hat{B}_{i1} = (X'_{i1}X_{i1})^{-1}X'_{i1}Y_{i1}$$

Then, the macro sum of squared residuals, S_e, can be written as:

$$S_e = \hat{E}'_{i1}\hat{E}_{i1}$$

$$= \hat{\delta}^{*'}_{ij1}Z^{*'}_{ij1}A'AZ^{*}_{ij1}\hat{\delta}^{*}_{ij1} + 2\hat{\delta}^{*'}_{ij1}Z^{*'}_{ij1}A'A\hat{U}_{ij1}$$

$$+ \hat{U}'_{ij1}A'A\hat{U}_{ij1} \tag{30}$$

where $\hat{\delta}^*_{ijl}$ and $\hat{U}_{ijl}$ are the OLS estimates of S^*_{ijl} and U_{ijl}, respectively. The first and third terms in the right-hand side of (30) are positive definite, while the second term is indefinite. As number of observations goes to infinity, the indefinite term is assumed to go to zero. However, this term is nonzero with finite samples which will have important implications for the relation between macro and micro CUSUM-SQ series. To see this, let PD and ID denote the first and second terms of (30), respectively. The third term can be written as:

$$\hat{U}'_{ijl}A'A\hat{U}_{ijl} = \sum_{l=1}^{Q}(\sum_{j=0}^{m-1}\hat{u}_{ijl})^2$$

$$= \sum_{l=1}^{Q}[\sum_{j=0}^{m-1}(\hat{u}_{ijl})^2 + 2\sum_{j=0}^{m-2}\sum_{d=j+1}^{m-1}\hat{u}_{ijl}\hat{u}_{idl}] \qquad (31)$$

Assuming the d^{th}-order serial correlation coefficient is the d^{th} power of the first-order serial correlation coefficient, i.e., $p_d = p^d$, then relation (31) can be written as:

$$\hat{U}'_{ijl}A'A\hat{U}_{ijl} = \sum_{l=1}^{Q}[\sum_{j=0}^{m-1}(\hat{u}_{ijl})^2 \ (1+2\sum_{d=1}^{m-1}(\frac{m-d}{m})P^d_{ul})] \qquad (32)$$

Under the stationarity assumption of the distributions of the micro residuals, u_{ijl}, the time subscript l can be dropped without loss of generality, and (32) becomes[11, 12]

$$\hat{U}'_{ijl}A'A\hat{U}_{ijl} = S_u \ (1 + 2\left[\frac{mP_u(1-P_u)-P_u(1-P^m_u)}{(1-P_u)^2}\right]) \qquad (33)$$

where S_u is the sum of squares of micro residuals in model (22).

Now, from (19), the macro CUSUM-SQ series is

$$C_r^e = S_{er} / S_{eQ} \tag{19}$$

If we substitute (33) into (30) and the resulting outcome into (19) we get

$$C_r^e = \left(1 + \frac{\Psi}{1+\Omega}\right) C_{mr}^u \tag{34}$$

where

$$\Psi = \frac{PD_r + ID_r}{(PD_Q + ID_Q)C_{mr}^u} - 1, \ \Omega = \frac{\hat{U}_{ij1}' A'A\hat{U}_{ij1}}{PD_Q + ID_Q}, \ \frac{\Psi}{1+\Omega} \geq -1, \ \lim_{r \to Q} \frac{\Psi}{1+\Omega} = 0,$$

and C_{mr}^u is the corresponding micro CUSUM-SQ series since this relation has reference only to micro periods m(K+1), m(K+2), ..., mQ.

In simple models where no lagged variables whatsoever are included the term $\frac{\Psi}{1+\Omega}$ is equal to zero and thus the macro CUSUM-SQ series is the same as the corresponding micro CUSUM-SQ series. However, as will be indicated later, the reduction in sample size resulting from temporal aggregation leads simultaneously to a decrease in the calculated test statistic and an increase in the tabulated critical value. This in turn increases the probability of accepting the null hypothesis of no structural change when it should be rejected, i.e., it increases type II error.

In model (22) where lagged exogenous variables are present, temporal aggregation results in the macro CUSUM-SQ series C_r^e being greater than, smaller than, or equal to the micro CUSUM-SQ series C_{mr}^u depending on whether $\frac{\Psi}{1+\Omega}$ is greater than, smaller than, or equal to zero. Then, $\frac{\Psi}{1+\Omega}$ can be regarded as the proportion of bias in the macro CUSUM-SQ series due to the effect of omitted (overlapping) variables and their interaction with micro residuals. If this interaction is equal to zero, i.e., if ID=0, the term $\frac{\Psi}{1+\Omega}$ would be negative, and C_r^e will be smaller than C_{mr}^u for all values of r.

From the previous section, calculated test statistics for both macro and (corresponding) micro CUSUM-SQ series are given respectively by:

$$C_c^e = \max \left| C_{K+2z}^e - \frac{z}{Z_e} \right|, \; z=1,2,\ldots,Z_e-1, \text{ and } Z_e=(1/2)(Q-K) \quad (35)$$

and

$$C_{mc}^u = \max \left| C_{m(K+2z)}^u - \frac{z}{Z_{mu}} \right|, \; z=1,2,\ldots,Z_{mu}-1, \text{ and } Z_{mu}=(1/2)(Q-\frac{K}{m}) \quad (36)$$

If no lagged variables are present, $C_{K+2z}^e = C_{m(K+2z)}^u$ as we indicated above. In this case C_c^e, the calculated macro test statistic, would always be less than C_{mc}^u, the calculated micro test statistic since by definition, $Z_e < Z_{mu}$.[13] This result will not necessarily hold in the presence of lagged exogenous variables because of the ID term in (30).

On the other hand, for given micro observational period, temporal aggregation results in fewer observations. This loss of information due to the decreased sample size is reflected in a higher tabulated critical value when testing the macro (in time sense) relationship for structural stability.[14]

To sum up this section, the analysis has shown, given some simplifying assumptions, that aggregation over commodities, unless the intercorrelation coefficients are independent of the number of micro residuals used in computations, may result in inconsistency between the micro and macro evidences of structural change in parameter vectors. That is, it is possible, given the same model specification, to find stable macro parameters and unstable micro parameters or vice versa unstable macro parameters and stable micro parameters. Temporal aggregation, under the assumptions of stationarity of micro residuals and of no lagged variables, results in identical micro and macro CUSUM-SQ series. However, the reduction in sample size caused by this type of aggregation leads simultaneously to a decrease in the calculated macro test statistic and an increase in the tabulated critical value with the implication that temporal aggregation tends to stabilize the estimate of the parameter vector over the observational period. When lagged exogenous variables are present, the micro and macro CUSUM-SQ series are no longer identical because of omitted (overlapping) variables and their interaction with the micro residual estimates. Possibly, this could result in a larger calculated macro test statistic. Yet, it is likely for this increase to be offset by the rise in the tabulated critical value. Hence, one could conclude that, in general, structural change is less evident when temporally aggregated data are used.

III. Estimation Model

Based on conventional maximization theory, the most widely used specification expresses the demand for imports of the i^{th} commodity as a function of one relative price ratio and an activity variable, all in period t.[15] That is,

$$m_{it} = f\,(p_{it}, y_{it}); \qquad i=1,2,\ldots,n;\; t=1,2,\ldots,T \tag{37}$$

where

m_{it} = physical quantity imported of the i^{th} commodity in period t,

p_{it} = ratio of pm_{it} to pd_{it}, where pm_{it} is foreign import price of the i^{th} commodity in period t expressed in domestic currency, and pd_{it} is price of a domestic substitute for the i^{th} commodity in period t, and

y_{it} = an activity variable, expressed in real terms, of the i^{th} commodity in period t.

This specification assumes that the full impact of a change, say, in the U.S. business activity level in period t is felt within the same period. However, lags are a ubiquitous feature of life. If the time interval between observations is so short that the influence of a change is distributed over several time periods it is more realistic, in this case, to assume that certain length of time is required for the influence of this change to become fully effective. In this paper, we assume that the relevant lag has a finite length. This assumption can be justified as follows. Several research efforts (e.g., Branson, 1968; Miller and Fratianni, 1974; and Rhomberg and Boissonneault, [illegible]5) have indicated that time lags between U.S. imports and their determinants appear to be short. This short lag may be due to the fact that importers take into account the lagged effect of the changes in causal variables when making their decisions to import, or because the underlying trade data from which price-quantity series are obtained are based on actual transactions. Hence, current trade flows are linked to their contracted prices rather than to the prices at which future trade flows are currently being contracted. Each price series, therefore, contains an implicit lag based upon the particular contracting horizon of the relevant commodity.

On the other hand, reparametrization of an infinite distributed lag model results in using lagged values of the dependent variable as a right-hand side explanatory variable. Since the design matrix is no longer non-stochastic, the CUSUM-SQ test is not applicable (Brown et al., 1975, p. 151). Moreover, Bopp and Pendley (1978, p. 115) state "[the reparametrization] leads to very imprecise long-term elasticity estimates; can allow for a reversal of the relative precision on short- and long-term price and income elasticities; and is particularly vulnerable to multicollinearity problems when long-term elasticities are estimated."

Hatanaka-Wallace Distributed Lag[16]

This type of form free, finite distributed lag, developed by Hatanaka and Wallace (1979), is an attempt to take advantage of positive serial correlation in economic data responsible for high collinearity among a contemporaneous variable and its lag. This is accomplished by estimating the moments of the lag distribution instead of its ordinates. These moments, which are linear transformations of short-run effects, will have, as shown by the authors (pp. 5-17), lower variance with the precision ordered from lower to higher moments.

The proposed transformation from short-run effects to moments is as follows. Let the distributed lag model be

$$ {}_{T}Y_{1} = {}_{T}X_{N+1}B_{1} + {}_{T}U_{1}, \tag{38} $$

where Y is a dependent column vector, X is a design matrix composed of contemporaneous and N lagged values of one independent variable, B is a short-run parameter vector, and U is a disturbance vector assumed to satisfy conditions of normality, homoscedasticity, and serial independence.[17] The moment vector U is related to the short-run parameter vector B by the transformation matrix Ŧ as follows:

$$ {}_{N+1}\mathit{U}_{1} = {}_{N+1}\text{Ŧ}_{N+1}B_{1}. \tag{39} $$

In scalar form, (39) gives the g^{th} moment as:

$$u_g = \sum_{h=0}^{N} h^g \, b_h, \qquad g,h=0,1,2,\ldots,N \tag{39a}$$

$\mathcal{F}$ is nonsingular since it is a bordered Vandermonde matrix. Hence, B can be written in terms of U. That is,

$$B = \mathcal{F}^{-1} U, \tag{40}$$

and the estimable model is, then, given by

$${}_TY_1 = X^{(N)}{}_{N+1}U_1 + {}_TU_1, \tag{41}$$

where

$$X^{(N)} = X\, \mathcal{F}^{-1}.$$

If the investigator is interested in only the first n+1 moments where n is less than N, Silver and Wallace (1980) show that both U and $\mathcal{F}$ are modified to:

$${}_{N+1}U_1 = \begin{bmatrix} {}_{n+1}U^*{}_1 \\ \cdots\cdots \\ {}_{N-n}B^*{}_1 \end{bmatrix}, \text{ and } {}_{N+1}\mathcal{F}_{N+1} = \begin{bmatrix} {}_{n+1}\mathcal{F}^*{}_{N+1} \\ \cdots\cdots\cdots \\ {}_{N-n}R_{N+1} \end{bmatrix}, \tag{42}$$

where U^* is the vector of the first n+1 elements of U, B^* is the vector of the last N-n elements of B, $\mathcal{F}^*$ is the first n+1 rows of $\mathcal{F}$, and

$${}_{N-n}R_{N+1} = ({}_{N-n}0_{n+1} \,\vdots\, {}_{N-n}I_{N-n}).$$

Modifying the Hatanaka-Wallace Lag

Although quite good precision is gained by such transformations, it is clear that the Hatanaka-Wallace technique as presented above does not overcome the problem of degrees of freedom encountered in distributed lag models.[18] This problem becomes more severe when we know that time-series data are used to estimate such models since most time series are not available in enough length. A proposed solution, used in this study, combines Hatanaka and Wallace lag with Almon lag (Almon, 1965) and takes advantage of the fact that the transformation matrix of the Almon lag is the transpose of the matrix $\mp^*$ in (42) if and only if the polynomial in the Almon lag is restricted to be of degree equal to n, the number of moments of interest to the investigator, where N, the number of lagged periods, is assumed the same.

The basic idea behind the Almon lag is to approximate the time path of the lag parameters with a polynomial. The number of parameters to be estimated depends only on the degree of the polynomial n, not on the length of the lag N. Therefore, if the degree is fairly low, there should be no problem with degrees of freedom. This can be seen from the following relation:

$$ {}_{N+1}B_1 = {}_{N+1}\mp^{*'}{}_{n+1}A_1, \tag{43}$$

where $\mp^{*'}$ is the Almon transformation matrix; and A is the coefficient vector of the approximating polynomial. In scalar form, (43) gives the h^{th} short-run effect as:

$$ b_h = \sum_{g=0}^{n} h^g a_g, \qquad g=0,1,2,\ldots,n;\; h=0,1,2,\ldots,N \tag{43a}$$

(Koutsoyiannis, 1977, pp. 299-304). Substituting (43) into (38), we get

$$ {}_{T}Y_{1} = {}_{T}X^{(A)}{}_{n+1}A_{1} + {}_{T}U_{1}, \tag{44} $$

where $X^{(A)} = X \mp^{*'}$. A comparison of (38) and (44) indicates that number of parameters to be estimated is reduced by N-n.

Now, the first n+1 moments are related to the N+1 short-run effects as follows:

$$ {}_{n+1}U^{*}{}_{1} = {}_{n+1}\mp^{*}{}_{N+1}B_{1}. \tag{45} $$

Since $\mp^{*}$ is singular, a <u>unique</u> solution for B in terms of U^{*} does not exist. This problem, however, can be solved by substituting (43) into (45). This results in (46):

$$ {}_{n+1}U^{*}{}_{1} = {}_{n+1}(\mp^{*}{}_{N+1}\mp^{*'})_{n+1}A_{1}. \tag{46} $$

The synthesis transformation matrix $(\mp^{*}\mp^{*'})$ is symmetric, and, thus, has an inverse.[19] A can uniquely be expressed in terms of U^{*} as follows:

$$ A = (\mp^{*}\mp^{*'})^{-1}U^{*}. \tag{47} $$

The substitution of (47) into (43) gives the following estimable model:

$$ {}_{T}Y_{1} = {}_{T}X^{(n)}{}_{n+1}U^{*}{}_{1} + {}_{T}U_{1}, \tag{48} $$

where $X^{(n)} = X \mp^{*'}(\mp^{*}\mp^{*'})^{-1}$.

The Model

Given the fairly standard specification in (37), the basic import demand equation in general lag form is given, using matrix notation, as:

$$_TM_1 = {_TP_{N_p+1}}B_{p_1} + {_TY_{N_y+1}}B_{y_1} + {_TU_1}, \tag{49}$$

where M is a vector of i^{th} commodity imports, P is a price matrix for the current price ratio and its N_p lags, B_p is the relevant parameter vector, Y is an activity matrix for the contemporaneous activity variable and its N_y lags, B_y is the relevant parameter vector, and U is a disturbance vector assumed $N(0,\sigma^2)$. If M, P, and Y are expressed in terms of logarithms, (49) is viewed as log-linear, otherwise it is linear.

Applying the modified version of Hatanaka-Wallace distributed lag, as outlined earlier, to (49) results in the following estimation model:

$$_TM_1 = {_TP^{(n_p)}}_{n_p+1}\mu^*_{p_1} + {_TY^{(n_y)}}_{n_y+1}\mu^*_{y_1} + {_TU_1}, \tag{50}$$

where $P^{(n_p)} = P\,\Psi^{*'}_p\,(\Psi^*_p\Psi^{*'}_p)^{-1}$, and $Y^{(n_y)} = Y\,\Psi^{*'}_y\,(\Psi^*_y\Psi^{*'}_y)^{-1}$. μ^*_p and μ^*_y are price and activity moment vectors of the relevant lag distributions, where $\mu^*_p < 0$, and $\mu^*_y > 0$.

IV. Empirical Results

The theoretical findings of Section III are examined in terms of a particular application, using monthly data about the U.S. imports of cocoa, natural rubber, and unmanufactured wool for the period from July 1959 to June 1979. From this first set of data two other series are constructed. One is constructed by aggregation over an interval of three months, i.e., from monthly to quarterly. Another is constructed by aggregation over the three commodities.[20]

With respect to the choice of the lag length, Hatanaka and Wallace (1979, p. 19) stated:

> If one were to go directly for lag moments in a new data set with little or no prior information about the form of the lag distribution, there is a problem of choosing N, the number of lagged terms to include. If N is too small, biased estimates are a result, and if N is too large, variances become a problem. An empirical approach would be to vary N, looking for stability in the lag moment estimators.

In applying this approach, we experimented with different lag lengths for both relative prices and the activity variable in each equation. The 'stabilizing' lag lengths were found to be 36 months for relative prices and 24 months for activity variable in the natural rubber and unmanufactured wool import demand equations. For the cocoa equation, no significant lag structure was found. However, the application of a stepwise procedure showed that a best fit (in terms of expected signs and significant estimates) can be obtained when contemporaneous relative prices, and activity variable lagged three months are used as explanatory variables in this equation.

In regard to the number of moments to be estimated, it was decided to estimate only the first three moments: the long-run effect (zero-order moment, u_0), the mean of lag distribution (first-order moment, u_1), and the variance of lag distribution (second-order moment, u_2).[21] The reasons behind this choice are: low-order moments are of considerable policy interest. The low-order moments can be estimated more precisely than higher-order moments (Hatanaka and Wallace, 1979, p. 5). This choice implies a polynomial of second degree. Experimentation undertaken by Hooper (1974) and Stern <u>et al</u>. (1979) has indicated that good results for the U.S. import demand equations are possible when a polynomial of second degree is used. Finally, even with estimation limited to the first three moments, higher-order moments still can be obtained. For example, the estimated moment vector for relative prices in the rubber equation $\hat{u}_p^* = (\hat{u}_0\ \hat{u}_1\ \hat{u}_2)'$, can be converted back to a short-run parameter vector, $\hat{B}_p = (\hat{b}_0\ \hat{b}_1\ \ldots\ \hat{b}_{36})'$, using the following relation:

$$\hat{B}_p = \mp_p^{*'} (\mp_p^* \mp_p^{*'})^{-1} \hat{u}_p^* \qquad (51)$$

Once obtained, the short-run effects can be used to compute any of the higher-order moments using relation (39a).

Equation (50) was estimated for the three commodities (cocoa, rubber, and wool) and their aggregate using the monthly and the constructed quarterly data, and the forward CUSUM-SQ test was run for each regression equation. Table 1 reports the calculated CUSUM-SQ statistics, where the expression in parentheses under the calculated CUSUM-SQ statistic indicates whether it is significant (S) or insignificant (NS), and gives the level of significance.[22]

Table 1 indicates the presence of structural change in the monthly equations of U.S. cocoa, rubber, and wool imports when these equations are fitted in a linear functional form. Strictly speaking, the null hypothesis, H_0, of no structural change is rejected at a significance level of 1 percent when the linear functional form is used. This null hypothesis, however, can not be rejected with the log-linear form being used. The wool equation is the only one that showed a pattern of behavior insensitive to the functional form used. For this equation, the evidence of structural change is significant at 1 percent irrespective of which functional form is used.

Table 1. Calculated CUSUM-SQ statistics for the monthly and quarterly equations of the U.S. imports of cocoa, natural rubber, unmanufactured wool, and their aggregate for the period July 1959-June 1979

Functional Form	Commodity	Monthly	Quarterly
Linear	Cocoa[a]	0.2109 (S at 1%)	0.2534 (S at 1%)
	Rubber	0.1705 (S at 1%)	0.1098 (NS at 20%)
	Wool	0.2827 (S at 1%)	0.2157 (S at 10%)
	Aggregate[b]	0.2178 (S at 1%)	0.1492 (NS at 20%)
Log-Linear	Cocoa[a]	0.0997 (NS at 10%)	0.[illegible]8 (NS at 20%)
	Rubber	0.0680 (NS at 20%	0.1355 (NS at 20%)
	Wool	0.2652 (S at 1%)	0.3328 (S at 1%)
	Aggregate[b]	0.1055 (S at 10%)	0.1247 (NS at 20%)

[a]The U.S. demand for cocoa was estimated using current relative prices, and an activity variable lagged three months (one quarter).

[b]It seems that the structure of the relative prices lag is distorted by commodity aggregation. Estimation with this lag results in a poor fit and unexpected signs. Good results are obtained when this lag is dropped out. Table 1 reports the results for the latter case. The reader should be cautious since specifications are no longer identical, which is a necessary condition to determine the pure effect of aggregation on parameter stability.

As far as this study is concerned, these results will serve as our criteria to evaluate the distortions introduced through temporal and/or commodity aggregation onto the structural stability of micro equations. The process of evaluation is carried out in the remaining part of this section.

Temporal Aggregation

The three equations for U.S. cocoa, rubber, and wool imports are reestimated, and the CUSUM-SQ test is applied one more time using quarterly data constructed from the monthly data as outlined in the appendix. The results are reported in Table 1.

A comparison of these quarterly results with the previous monthly results suffices to indicate that when temporal aggregation is applied to the rubber and wool equations fitted in a linear form, structural change becomes less evident (the case for wool equation) or, more strikingly, turns to structural stability (the case for rubber equation). In the latter case, structural change, which was significant at 1 percent becomes insignificant even at a 20 percent level of significance.

With the wool equation fitted in a log-linear form, the calculated CUSUM-SQ statistic has increased relative to the corresponding micro statistic, and this increase has more than offset the increase in the tabulated statistic caused by the decline in the degrees of freedom, as a result of temporal aggregation, from 198 to 62.[23] With the effect of temporal aggregation on structural change almost cancelled out, the same inference about the null hypothesis of structural stability is made for the quarterly log-linear equation of wool imports. Structural change in the quarterly equation is, as in the monthly equation, significant at 1 percent.

Despite the fact that the calculated CUSUM-SQ statistic is doubled by temporal aggregation of the rubber equation; the structural change is still insignificant at 20 percent, which implies that the increase in the tabulated critical value as the number of degrees of freedom declined from 198 to 62 has more than offset the rise in the calculated statistic.

The situation is different for the cocoa equation. The structural change previously found insignificant at 10 percent in the monthly log-linear equation is now insignificant at 20 percent in the quarterly equation, while the same inference about the null hypothesis of no structural change is made in the quarterly as in the monthly equation when the linear functional form is used.

Commodity Aggregation

To examine how aggregation over cocoa, rubber, and wool imports affects the evidence of structural change obtained for the micro equations, an aggregated time-series is constructed as outlined in the appendix. This constructed series is then used to estimate the aggregative equation and to apply the CUSUM-SQ test of structural change. The result is presented in Table 1 under the heading "Monthly."

For our particular application, this result shows that commodity aggregation, in comparison with temporal aggregation, has no significant distorting impact on the evidences of structural change previously obtained for the micro equations. Precisely speaking, when the linear functional form is used, the evidences of structural change which were significant at 1 percent for the monthly cocoa, rubber, and wool equations are transmitted as completely as possible to the aggregative equation where the underlying macro parameter vector is evidently unstable at a 1 percent level of significance.

When the log-linear form is used, the evidences of structural change in the monthly cocoa, and rubber equations were insignificant at 10 and 20 percent, respectively, while for the monthly wool equation, structural change was significant at 1 percent. This mix of evidence is averaged in the aggregative equation where structural change is significant at 10 percent.

Simultaneous Aggregation

By 'simultaneous' aggregation is meant the application of both aggregation over commodities and aggregation over time periods to the monthly series of cocoa, rubber, and wool imports. With the help of this constructed series, a quarterly aggregative equation is estimated, and the CUSUM-SQ test is applied. The result is provided in Table 1 under the heading "Quarterly."

The result indicates that the null hypothesis of parameter stability cannot be rejected even when a significance level as high as 20 percent is used. It is true that this evidence is in contradiction with the quarterly evidence obtained for the cocoa equation fitted in a linear form and wool equation fitted in a log-linear form, where the same null hypothesis is rejected at a 1 percent level of significance. However, a comparison with the monthly aggregative equation, in Table 1, reveals that temporal aggregation, and not commodity aggregation, is largely responsible for this result.

V. Summary and Conclusions

It has been the intention of this study to analyze the common practice of using highly aggregated data when testing for the presence of structural change in the parameter vector underlying U.S. import demand function. In this study, only the cases for aggregation over commodities and aggregation over time periods are considered.

On a more general analytical level, we have shown that commodity aggregation may distort the structural stability (instability) of the micro relations especially if the intercorrelation coefficient for each pair of commodities is not independent of the number of micro residuals used in the computation of this coefficient. As a result of this distortion, unstable (stable) micro parameters may underlie stable (unstable) macro parameters.

Implicit in the analysis was the assumption that number of commodities over which aggregation takes place is constant. This assumption is obviously unrealistic whenever a period of many years in a rapidly changing economic system is investigated. It is evident that a change of such a number implies that the set of micro parameters of the equations which are dropped because their commodities have disappeared will also disappear. In the absence of this assumption, a natural outcome would be an increased uncertainty in regard to the influence of commodity aggregation on stability (instability) of the micro parameters, since any thing can happen.

The analysis also has shown that aggregation over time periods of a micro (in time sense) model containing no lagged variables and with stationary residuals, results in no distortions with respect to the

CUSUM-SQ series. The macro CUSUM-SQ series are exactly the same as the 'corresponding' micro series. However, the reduction in sample size caused by temporal aggregation leads simultaneously to a decrease in the calculated and an increase in the tabulated test statistics. The result is to reject the null hypothesis of no structural change less often.

With lagged exogenous variable present, we have demonstrated that aggregation over time periods, by omitting the overlapping variables from the macro relations, may result in a larger calculated macro statistic, relative to the calculated micro statistic. Nevertheless, it is likely for this increase to be offset by the rise in the critical value of the test statistic due to the reduction in sample size as pointed above.

On a more specific empirical level, we have found supporting evidence for the previously mentioned theoretical findings, where with temporal aggregation from monthly to quarterly, structural change in the U.S. import demand equations for cocoa, rubber, and wool became less evident, and where aggregation over commodities tended to average the micro evidence of structural change.

One thing is worth noting. Our empirical estimation and testing have been done for both the linear and log-linear functional forms. A comparison of the results under each functional form reveals that the evidence of, and influence of aggregation on, structural change are both sensitive to the functional form used.

The conclusions of this study are as follows:

(a) For the sample of commodities examined, aggregation over time periods yields estimates of import demand parameters that show more structural stability. Temporally aggregated data is thus less apt to show unstable import demand functions.

(b) The sample of raw material commodities shows no evidence, for log-linear functions, of serious aggregation bias when all commodities are aggregated.

(c) Two specific functional form for demand functions were examined, linear and log-linear. Some differences in stability inferences were observed between these different specifications. Further work on this problem is required.

Overall, the conclusions would be that aggregation questions as well as functional form choice should be considered when examining stability issues. Certainly if both aggregated and disaggregated data sets are available, both should be examined.

Footnotes

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[1]Recursive estimation in a linear regression model is introduced by Plackett (1950). However, as pointed out by Farebrother (1978), recursive residuals have been known since 1891.

[2]This is the special case that Brown *et al*. (1975) have preferred to deal with, and which we also assume. For the case when errors are serially dependent, see McGilchrist and Sandland (1979).

[3]This can be seen as follows (Mendenhall and Schaeffer, 1973, p. 408):

$$V(v_r)=V(y_r)+V(x_r\hat{B}_{r-1})-2COV(y_r,x_r\hat{B}_{r-1}).$$

Since y_r was not employed in the calculation of $\hat{B}_{r-1}$ and that, in fact, it was randomly selected and hence independent of $x_r\hat{B}_{r-1}$, it follows that the covariance of y_r and $x_r\hat{B}_{r-1}$ is equal to zero. Then,

$$V(v_r)=V(y_r)+V(x_r\hat{B}_{r-1})$$

$$=\sigma_u^2+\sigma_u^2\,[x_r(X'_{r-1}X_{r-1})^{-1}x'_r]=\sigma_u^2\,[1+x_r(X'_{r-1}X_{r-1})^{-1}x'_r]$$

where σ_u^2 is the variance of the original residuals of (3). Thus, for the recursive residuals w_r to have variance equal to σ_u^2, v_r has to be divided by the square root of $1+x_r(X'_{r-1}X_{r-1})^{-1}x'_r$.

[4]The discussion will be in terms of forward recursion. Backward recursion is similar, however, the former is generally more useful (Evans, 1977, p. 11). Schweder (1976, p. 494) advocates the backward recursion on the basis that it picks up real effects of structural change, if there are any, from the beginning.

[5] Farely *et al*. (1975, pp. 305-306) have pointed out that the CUSUM-SQ test does not have an asymptotic power of unity as number of observations goes to infinity. However, simulation analysis by Garbade (1977, p. 57) has shown that the power of this test increases as a function of the magnitude of the instability, but does not grow monotonically as the number of observations increases.

[6] The CUSUM-SQ test is an approximate test. The approximation lies in using the distribution appropriate to a continuous process (Riddell, in Cameron, 1979, p. 262n). Hence, the test would give significant results more often than the exact test would give, but the discrepancy is small if $T-K > 30$ (Brown *et al*., 1975, p. 155).

[7] A similar relation was derived by Grunfeld and Griliches (1960, p. 10)

[8] This follows directly from the first assumption above (see p. 8).

[9] Aggregation here is assumed to take place over time periods such that each macro time unit contains m micro time units.

[10] Since it is easy for the reader to check the truth of this statement for conditions (a) and (b), a proof is provided here only for condition (c). For this condition, $E_{i1} = (I_Q - X_{i1}W_{i1})AX_{ij1}B_{ij1} + AU_{ij1}$ where $W_{i1}=(X'_{i1}X_{i1})^{-1}X'_{i1}$. From (3.3.1) and (3.3.2), $X_{ij1}B_{ij1}=Z_{ij1}\delta_{ij1}+Z^*_{ij1}\delta^*_{ij1}$, and from (3.3.3) $X_{i1}=AZ_{ij1}$, then $E_{i1}=(I_Q-X_{i1}W_{i1})AZ^*_{ij1}\delta^*_{ij1}+AU_{ij1}$ since $(I_Q-X_{i1}W_{i1})X_{i1}=0$, but $B_{i1}=\delta_{ij1}$ requires that $W_{i1}AZ^*_{ij1}\delta^*_{ij1} = 0$. This can be shown as follows. From condition (c), $B_{i1}=W_{i1}AX_{ij1}B_{ij1}$. After substitution for $X_{ij1}B_{ij1}$, B_{i1} is given as: $B_{i1}=W_{i1}AZ_{ij1}\delta_{ij1}+W_{i1}AZ^*_{ij1}\delta^*_{ij1}$. However, $W_{i1}AZ_{ij1}=W_{i1}X_{i1}=I$ then for B_{i1} to equal δ_{ij1}, $W_{i1}AZ^*_{ij1}\delta^*_{ij1}$ must be set equal to zero. This completes the proof.

[11]Stationarity of v_r means that $E(v_t^2)$ and $E(v_t v_{t-s})$ are independent of t, i.e., variance v_t is the same for all t and covariance between v_t and v_{t-s} is a function only of the distance s and not of the starting point. $E(v_t v_{t-s})$ is called the lagged covariance with lag = s.

[12]This relation is obtained as follows:

$$\sum_{d=1}^{m-1}(m-d)P_u^d = m\sum_{d=1}^{m-1}P_u^d - \sum_{d=1}^{m-1}dP_u^d$$

$$= m[(P_u-P_u^m)/(1-P_u)] - [(\sum_{d=1}^{m-1}P_u^d - (m-1)P_u^m)/(1-P_u)]$$

$$= m[(P_u-P_u^m)/(1-P_u)] - [(\{(P_u-P_u^m)/(1-P_u)\} - (m-1)P_u^m)/(1-P_u)]$$

$$= (mP_u(1-P_u)-P_u(1-P_u^m)) / (1-P_u)^2.$$

[13]Although this result has been derived using only the corresponding micro CUSUM-SQ series, it pertains to all other values. This is because if C_{mc}^u is not the actual calculated micro test statistic, there will be another value, call it C_c^u, which is greater than this value. $C_c^u = \max | C_{K+2z}^u - \frac{z}{Z_u} |$, $z=1,2,\ldots,Z_u-1$, and $Z_u = 1/2(mQ-K)$. C_{mc}^u is based on a subset of the set on which C_c^u was based. Since C_c^u is the global maximum which may or may not correspond to the local maximum C_{mc}^u, $C_{mc}^u \leq C_c^u$.

[14]Rowe (1976, p. 752n) has suggested extending the period of analysis until the corresponding sample size is obtained. However, such an extension would lead to noncomparable results since the observational periods and hence the informational contents are not the same.

[15]See, e.g., Houthakker and Magee (1969), Goldstein and Khan (1976), Khan (1974, Kreinin (1973), Kwack (1972), Miller and Fratianni (1974), and Yadav (1975 and 1977).

[16]We are indebted to Professor Thomas Johnson of NCSU for bringing this type of lag distribution to our attention.

[17]The assumption of only one independent variable implies no loss of generality.

[18]Note that both Ŧ and Ŧ̄ have the same dimensions, (N+1)x(N+1). That is, even if the investigator is interested in only the first n+1 moments, he still has to estimate useless N-n parameters. Moreover, N-n df's are lost for these useless parameters.

[19]For this to be the case, collinearity among columns should be absent. $Ŧ^{*'}$ will always satisfy this condition.

[20]For aggregation technique and data description, see the appendix.

[21]In fact, these u's are unadjusted moments in a non-normalized lag distribution. The moments of final interest would require the following transformation. $M_1=u_1/u_0$, and $M_2=u_2/u_0 - M_1^2$, where M_1 is the mean of the lag distribution, and M_2 is its variance (Hatanaka and Wallace, 1979, p. 5).

[22]Serial correlation was present in the cocoa, wool, and aggregative equations. The evidence of structural change did not change as we adjust for first-order serial correlation in those equations. The exception is the linear quarterly wool equation where the evidence was significant only at 10 percent before adjustment and became significant at 2 percent after the adjustment. For this, the after-adjustment results are not reported.

[23]For df's=198, the tabulated statistic is 0.15631 (Durbin's table is entered at $1/2\alpha=0.005$, and n=98). For df's=62, this statistic is 0.26772 (Durbin's table is entered at $1/2\alpha=0.005$, and n=30).

Appendix

Data Description

Data is, and has always been, the chief stumbling block in any empirical study. Whether empirical results support or refute the theoretical findings depends heavily upon the data employed. Here, a brief description of the data series used in this study is given. The data are obtained from both Commodity Year Book and Survey of Current Business.

Since it would be impractical to attempt research on all commodities, the present study limits itself to only <u>three</u> commodities: cocoa, natural rubber, and unmanufactured wool. These commodities have been chosen arbitrarily and not for particular reasons. The data collected are monthly, seasonally unadjusted observations. It spans 20 years, from July 1959 through June 1979, and includes the following variables:

1. U.S. Cocoa Imports:

QMC - U.S. imports of cocoa (including shells) in thousands of pounds. The series is originally stated in thousands of long tons (long ton = 2240 pounds).

PMC - Spot cocoa bean price (ACCRA) in New York in cents per pound.

WPI_F - Seasonally unadjusted spot market price index for foodstuffs (1967 = 100).

SALES - Seasonally unadjusted shipments of food and kindred products in millions of U.S. dollars.

2. U.S. Natural Rubber Imports:

QMNR - U.S. imports of natural rubber (including latex and guayule) in thousands of pounds. The original series is stated in thousands of long tons.

PMNR - Average spot crude rubber price (smoked sheets) in New York in cents per pound.

WPI_RAW - Seasonally unadjusted spot market price index of raw industrials (1967 = 100).

CONSUMNR - Consumption of natural rubber in the United States in thousands of pounds. The original series is stated in thousands of long tons up to 1974 and from 1975 on in thousands of metric tons (metric ton = 0.984307 long tons).

3. U.S. Unmanufactured Wool Imports:

QMW - U.S. imports of unmanufactured wool (clean yield) in millions of pounds.

PMW - Average wool price in U.S. mills - ¢ lb.

WPI_TEX - Seasonally unadjusted wholesale price index of textile products and apparel (1967 = 100).

CONSUMW - Consumption of apparel wool in the United States in millions of pounds.

4. Other Variables:

POP - U.S. population including armed forces overseas.[1]

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[1]All variables used in regression equations are deflated by population in order to induce homoscedasticity of the error terms (Clements, 1978, p. 363).

From these raw data, the following endogenous variables m_i, and the exogenous variables p_i and y_i, i=c,r,w, are constructed as follows:

Commodity	Imports per capita	Relative prices	Activity variable per capita
Cocoa	m_c=QMC/POP	p_c=PMC/WPI_F	y_c=(SALES/WPI_F)/POP
Rubber	m_r=QMNR/POP	p_r=PMNR/WPI_RAW	y_r=CONSUMNR/POP
Wool	m_w=QMW/POP	p_w=PMW/WPI_TEX	y_w=CONSUMW/POP

Aggregation Technique

The fulfillment of this study's goal requires the construction of both commodity and temporal aggregates. These aggregates are constructed as follows:

1. Commodity Aggregates:

Let p_{ijl} and m_{ijl} be the price ratio and quantity imported, respectively, of the i^{th} commodity in month j of quarter l. Further, let $v_{ijl} = p_{ijl} m_{ijl}$, i=1,2,3; j=1,2,3; and l=1,2,...,80. Then, commodity aggregates, m^c_{jl}, p^c_{jl}, and y^c_{jl}, are defined, respectively, as:

$$m^c_{jl} = 3 \sum_{i=1}^{3} m_{ijl} \frac{v_{ijl}}{\sum_{i=1}^{3} v_{ijl}} ,$$

$$p^c_{jl} = \frac{\sum_{i=1}^{3} v_{ijl}}{m^c_{jl}} , \text{ and}$$

$$y^c_{jl} = 3 \sum_{i=1}^{3} y_{ijl} \frac{v_{ijl}}{\sum_{i=1}^{3} v_{ijl}} ,$$

where m^c_{jl}, p^c_{jl}, and y^c_{jl} are the quantity, price ratio, and activity variables, respectively, of U.S. aggregate imports of cocoa, rubber, and wool.

2. Temporal Aggregates:

Let $v_{i1} = \sum_{j=1}^{3} p_{ij1} m_{ij1}$, where p_{ij1} and m_{ij1} are defined as before. Then, the following relations are used to obtain quarterly observations from monthly data:

$$m_{i1}^{t} = \sum_{j=1}^{3} m_{ij1},$$

$$p_{i1}^{t} = \frac{v_{i1}}{m_{i1}^{t}} = \sum_{j=1}^{3} p_{ij1} \frac{m_{ij1}}{\sum_{j=1}^{3} m_{ij1}}$$

$$y_{i1}^{t} = \sum_{j=1}^{3} y_{ij1},$$

where m_{i1}^{t}, p_{i1}^{t}, and y_{i1}^{t} are the quarterly observations of quantity imported, price ratio, and activity variable, respectively, for the i^{th} commodity in quarter 1.

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