An Application of Markovian Analysis to the budgeting of the Public Service Sector in Egypt.

By

Dr. Samir M. El Gazzar

College of Commerce

Tanta University

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ABSTRACT

This paper applies the techniques of Markoving Analysis and Parts Explosion Matrices to the field of budgeting in the public service in Egypt. The expanded role of the Egyptian Government in public service necessitates the use of advanced planning and control systems for efficient utilization of available facilities and allocation of state funds for the public service. The techniques analyzed in this paper provide necessary information for budgeting and service flows. Two illustrations are presented for application in the case of education and health.

1. INTRODUCTION

The role of the Egyptian Government in the public service has been expanding over the years. Education, health, and public service have been receiving great consideration in the country's total budget. Statistics show the significant growth of the service in these fields.[1] In the field of education, the total number of students in public schools increased from 848,317 in 1977/78 to 1,252,629 in 1982/83. Total expenditure on public education for 1982/83 on both current and capital accounts was L.E. 975 million, while university education expenditure amounted to L.E. 617 million. Egypt also has developed a comprehensive public health service, which is extended mostly without charge. The government expenditure on health service in 1982/83 was L. E. 434 million. Social welfare service to citizens has been receiving great attention, and as a result has been obtaining the necessary economic resources to improve the standard of living. Welfare service includes assistance to special categories of the population who suffer from mental or physical handicaps.

The achievement of high standard of social service is not only dependent on the allocation of funds, but also on the efficient and effective utilization of available economic resources and facilities. Annual budgets and programs of operations in the

1. Annual Statistical Abstract, Ministry of planning, Central Statistical Office, Volume XX, Cairo, Egypt, 1983.

public services sector provide a tool of planning and control. The techniques analyzed in this paper provide the necessary information for budgeting and service flows. These techniques can be expanded to provide the necessary information for long run planning, expansion, or budget cuts.

It should be noticed that these techniques are applicable for budgeting and control in production sector too. The technique presented here is similar to « material requirements planning » systems that were developed by management scientists many years ago.

The remainder of this paper is analyzed as follows : Section II presents the type of information needed for budgeting in public service organizations. Section III includes the analysis of the technique and two illustrations in the fields of education, and health. Section IV presents some conclusions and recommendations for further research.

II. Service Organizations : Budgeting Information

In the conventional budgeting process, once the sales[2] forecasts are made (using probabilistic or deterministic models) the supporting schedules of production, purchases, labor, and supporting facilities can easily be prepared. In service organizations like schools, hospitals, and social welfare programs, services are provided to the recipient during an interval of time, and in different moving stages. For example, in high school, a student receives education (normally) in a three year period. During that period he (or she) is moving into progressive stages. These two characteristics of services flows, i.e., duration and moving states, require different types of information for budgeting. In some cases the span of time is standard, as in the field

2. In some cases, and because of capacity limitations, the budgeting process starts by production budget.

of education, but in other cases the span of time is determined at random and depends on indivivdual eases as in the case of in-patient hospital treatment.

In an educational institution the following information is needed for budgeting :

— what would the long-run number of students attending the institution be in any given year and what years would they be in ?

— What number of students would graduate each year and how many would fail ?

— Given a specific number of students in attendance now, how many students will graduate at the end of this coming year ?

The Markov steady state «absorption matrix» helps in answering these questions.[3] We shall assume familiarity with the fundamentals of Markov chain.[4] After determining the number of students enrolled in the different stages at time «t», for instance next year, the preparation of the service factors budgets is in order. The «parts explosion» matrix helps in preparing these supporting schedules. The parts explosion matrix provides us with the rates of requirements of the providing factors for each student/year. Providing factors in an educational institution include : teachers, rooms, administration, ... ets.

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3. The model can be adjusted to fit the growth state, or nonsteady states.
4. You can refer to some textbooks that discuss Markov chains: Derman, C. **«Finite State Markovian Decision Process,»** Academic Press, New Oork 1970; OR Cyert, R.N. et al. «Estimation of the Allowance for Doubtful Accounts by Markov Chains,» **Management Science,** Vol. 8, No. 3, 1962.

III. ANALYSIS : the Case of Education

To simplify calculations[5], assume a graduate school of two academic years has been experiencing the absorbing Markov chain below.

$$
\begin{array}{c} \\ P \\ F \\ Y_1 \\ Y_2 \end{array}
\begin{array}{c}
\begin{array}{cccc} P & F & Y_1 & Y_2 \end{array} \\
\left[\begin{array}{cc|cc}
.1 & .0 & .0 & .0 \\
.0 & .1 & .0 & .0 \\
\hline
.2 & .1 & .2 & .5 \\
.7 & .1 & .0 & .2
\end{array}\right]
\end{array}
=
\left[\begin{array}{c|c}
I & O \\
\hline
K & M
\end{array}\right]
$$

where :

P = Pass

F = Fail

Y_1 = First Year

Y_2 = Second Year

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ = is the identity matrix

$0 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ = is the null matrix

M = is the matrix of movements from first year to second year.

K = is the matrix of passing and failing in specific year (Y_1, Y_2).

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5. The use of computer, in real cases, solves the problem of the big calculations and makes the analysis applicable even with a matrix of 100 × 100 variables, or more.

The steady state, or long-run predications are based on the «fundamental matrix», A.

$$A = (I-M)^{-1} \qquad\qquad (I-M)^{-1}$$

By substitution :

$$A = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} .2 & .5 \\ 0 & .2 \end{bmatrix} \right\}^{-1}$$

$$= \begin{bmatrix} .8 & -.5 \\ 0 & .8 \end{bmatrix}^{-1} = \frac{\begin{bmatrix} .8 & .5 \\ 0 & .8 \end{bmatrix}}{.64}$$

Assuming that the graduate program admitted 1,000 students to begin with and admits 1,000 students per year for the following years, the questions raised at the beginning can be easwered now :

1. The long run unmber of students attending that school and year of attendance of each are calculated as follows ;

$$XA = (\,1{,}000, \;\; 0\,) \frac{\begin{pmatrix} .8 & .5 \\ 0 & .8 \end{pmatrix}}{.64}$$

$$= (1250{,}781)$$

Which says that in the long-run there will be 1250 students attending the first year of school, and 781 students attending the second year of school.

2. The number of students graduating and failing each year, given long-run attendance :

$$XAK = (1000,0) \frac{\begin{pmatrix} .8 & .5 \\ 0 & .8 \end{pmatrix}}{64} \begin{pmatrix} .2 & .1 \\ .7 & .1 \end{pmatrix} \text{ OR}$$

$$= (1250,\ 781) \begin{pmatrix} .2 & .1 \\ .7 & .1 \end{pmatrix} = (797,203)$$

which says that the long-run predictions of the number of students graduating from the program each year is 797 students, while 203 students are expected to fail.

3. Given a current enrollment, with distribution between Y_1 and Y_2 as (1200,900), then the expected number of graduates at the end of current year is :

$$(1200,900) \begin{pmatrix} .2 \\ .7 \end{pmatrix} = 870$$

The above nanlysis assumes the steady state condition, where the school has a specific capacity and the annual enrollment is stable. However, the chain can be revised to allow for changes in the probabilities of passing and failing. Also, the matrix, X, enrollment matrix, can be altered to reflect the desired growth or reduction in the admission policy.

Another usage of the above predicitions is the estimation of tuition receipts, especially if tuitions are based on semester of attendance (semester) and load of courses. Novertheless, this dimension is not important in Egypt since education on different levels is provided free.

Scheduling of Resources

After reaching predictions of student enrollment and their distribution, the scheduling of resources becomes the next order.

Economic resources needed are : Faculty, staff, classrooms, other facilities like library and computer equipments. The «parts explosion» matrix provides the rates of requirements from the providing factors for each student per academic year.

The construction of the parts explosion matrix depends on the description of the program of study. To illustrate, assume the school under consideration offers majors with the following requirements :

— 5 core courses and 3 elective courses must be taken by each student each year.

— on average, the enrollment of the core Course is 40 students while the enrollment of elective is 20 students.

— each course requires a room and a professor.

each professor teaches 4 courses per year.

— a room can be used 6 times a day, for 5 days a week for 2 semesters.

— a professor requires an office 1/3 the size of a classroom.

Using the above conditions, the parts explosion matrix can be constructed as follows :

$$
\begin{array}{c|ccccc|}
 & F_1 & F_2 & F_3 & F_4 & F_5 \\
F_1 & 0 & 0 & 0 & 0 & 0 \\
F_2 & \frac{1}{3} & 0 & 0 & 0 & 0 \\
F_3 & \frac{1}{240} & \frac{1}{80} & 0 & 0 & 0 \\
F_4 & \frac{1}{480} & \frac{1}{160} & 0 & 0 & 0 \\
F_5 & 0 & 0 & 3 & 5 & 0
\end{array} = F
$$

where :

F_1 = Rooms

F_2 = Professors

F_3 = Elective Courses

F_4 = Core Courses

F_5 = Major

F_{21} = represents the requirements of a professor from rooms.

$$\left(\frac{1}{3}\right).$$

F_{31} = represents the needs of the student taking an elective course from rooms, (20) students X 6 times X 2 semesters)

F_{32} = represent the needs of a student taking an elective course from professors (20 students X 4 professors).

F_{41} = represent the needs of a student taking a core course from rooms, (40) students X 6 times X 2 semesters).

F_{42} = represent the needs of a student taking a core course from professors, (40 students X 4 professors).

The total services consumption (TS) equals the internal consumption, XF, plus the external consumption, E. Hence.

$$TS = XF + E$$

$$= E\,(I-F)^{-1}$$

$$(I-F)^{-1} = I - F$$

$$\begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{vmatrix} - \begin{vmatrix} 0 & 0 & 0 & 0 & 0 \\ \frac{1}{3} & 0 & 0 & 0 & 0 \\ \frac{1}{240} & \frac{1}{80} & 0 & 0 & 0 \\ \frac{1}{480} & \frac{1}{160} & 0 & 0 & 0 \\ 0 & 0 & .3 & .5 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ -.333 & 1 & 0 & 0 & 0 \\ -.00416 & -.0125 & 1 & 0 & 0 \\ -.00208 & -.00625 & 0 & 1 & 0 \\ 0 & 0 & -.3 & -.5 & 1 \end{vmatrix}^{-1} = \begin{vmatrix} 1 & 0 & 0 & 0 & 0 \\ .333 & 1 & 0 & 0 & 0 \\ .00416 & .0125 & 1 & 0 & 0 \\ .00208 & .00625 & 0 & 1 & 0 \\ 0.0921 & 0.0335 & .3 & .5 & 1 \end{vmatrix}$$

E = the parameters matrix of the internal use of the providing factors, the steady state number of students in enrollment as calculated before is 2031 i.e., 1250 + 781. That number represents the enrollment of the major consideration, i.e., F_{55}. Assume also, the school needs 15 additional rooms for administration use, and 7 full time research professors.

$$E = [\,15,\ 7,\ 0,\ 0,\ 2031\,].$$

TS = (15, 7, 0, 0, 2031)

1	0	0	0	0
.333	1	0	0	0
.00416	.0125	1	0	0
.00208	.00265	0	1	0
.0921	.0335	3	5	1

(Rooms, profs, electives, core, majors)

= (205, 75, 6093, 10,155, 2031)

III.2 Analysis : The Case of Mental-Sickness Centers.

Assume the following hypothertical mental health center.

The center has a capacity of 1000 patients in terms of beds and facilities. The mental health care treatment is provided in two successive stages. Each stage represents a course of treatment, Moving from the first stage to the second is contingent on passing the first. Those who are not recovered by the end of the stage repeat the course of treatment in the same stage again. Each stage has a time length of 6 months.

Starting by 500 patients at (t=0), the number of admission at (t=1) depends on the number of patients that will pass stage 1. From the historical records of the center, management was able to build the following matrix information :

	R	NR	S1	S2
R	1	0	0	0
NR	0	1	0	0
S1	.2	.1	.2	.5
S2	.7	.1	.0	.2

where :

R = Recovered

NR = Not recovered.

S_1 = Stage one (represent course of treatment No. 1.)

S_2 = Stage two (represent course of treatment No. 2).

Let us define the following matrices ;

M = the matrix of movements betwen stages.

$$M = \begin{array}{c} \\ S_1 \\ S_2 \end{array} \begin{array}{c} \begin{array}{cc} S_1 & S_2 \end{array} \\ \begin{pmatrix} .2 & .5 \\ .0 & .2 \end{pmatrix} \end{array}$$

K = is the matrix of recovery and not recovery conditions.

$$K = \begin{array}{c} \\ R \\ NR \end{array} \begin{array}{c} \begin{array}{cc} R & NR \end{array} \\ \begin{pmatrix} .2 & .1 \\ .7 & .1 \end{pmatrix} \end{array}$$

$$N = (I—M)^{-1}$$

$$= \left[\begin{pmatrix} .1 & 0 \\ .0 & 1 \end{pmatrix} - \begin{pmatrix} .2 & .5 \\ .0 & .2 \end{pmatrix} \right]^{-1}$$

$$= \begin{pmatrix} .8 & .5 \\ .0 & .8 \end{pmatrix}^{-1} = \frac{\begin{pmatrix} .8 & .5 \\ .0 & .8 \end{pmatrix}}{.64}$$

From the conditions stated above, information regarding the following aspects can be obtained on an ex-ante basis :

— The long-run number of patients receiving treatments in the center.

— The number of recovered patients (and not recovered patients) in each stage and for each period.

— Given a state of enrollment say (N_1, N_2) what would be the expected number of recoveries at end of (t) and how many patients the center may admit next period ($t + 1$).

1) The permanent number of patients in the center (the steady state residency) is :

$$XN = (500,0) \underbrace{\begin{pmatrix} .8 & .5 \\ 0 & .8 \end{pmatrix}}_{.64} = (625,391)$$

2) The number of patients recovered and who repeat the course treatments each period can be determined using matrix K (the probability of R, NR, each stage), and the steady state number of attendance in each state, i.e., XNK matrix :

$$= (500,0) \underbrace{\begin{pmatrix} .8 & .5 \\ 0 & .8 \end{pmatrix}}_{64} \begin{vmatrix} .2 & .1 \\ .7 & .1 \end{vmatrix}$$

$$= (625,391) \begin{vmatrix} .2 & .1 \\ .7 & .1 \end{vmatrix}$$

$$= [\ (125 + 273),\ (63 + 39)\ ] = [\ 398,102\ ]$$

3) For any specific period where there are a specific number of residency patients, management can determine the number of expected recovered patients by the end of the year. Hence they can determine how many patients they will admit next period and how many excess demand they have ? Assume

at (t=3) the center has (600,400) in attendance. How many patients are expected to be recovered by end of period (3), and how many they can admit at beginning of (t=4) ? Expected recovery at end of (t=3).

$$= (600,400 \begin{pmatrix} .2 \\ .7 \end{pmatrix} = (120 + 280) = 400$$

Then the number of new admissions should not excess 400 patients, (assuming all beds are occupied at (t = 3)).

Scheduling of Resources

Let us assume the course of sickness remedy calls for 3 courses of psychiatry treatments and two courses of communication and skills development. Patients reside in the center during the treatment period. Psychiatry treatments (courses) are provided in groups of 5 patients, and the psychiatrist load per period is 2 groups. Communications and skills development classes are of 10 patients each, and the consultant load is 3 groups a period. Residency is 4 beds a room. The psychiatrist needs an office which is 1/2 room and so the consultant.

Define :

Z_1 = space of the center measured in rooms

Z_2 = psychiatrists

Z_3 = consultants and supervisors for communication and skills development.

Z_4 = psychiatry treatment per patient

Z_5 = communication and skills.

Z_6 = residency

Then, from the conditions set up, the parts explosion matrix is :

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
Z_1	0	0	0	0	0	0
Z_2	1/2	0	0	0	0	0
Z_3	1/2	0	0	0	0	0
Z_4	1/5	1/10	0	0	0	0
Z_5	1/10	0	1/30	0	0	0
Z_6	1/4	0	0	3	2	0

$= Z$

Total services consumption = Internal consumption (XZ) + External consumption (E), Hence :

$$TS = XZ + E. \qquad \text{OR } E\,(I - Z)^{-1}$$

$(1-Z)^{-1} =$

	Z_1	Z_2	Z_3	Z_4	Z_5	Z_6
Z_1	1	0	0	0	0	0
Z_2	—.5	1	0	0	0	0
Z_3	—.5	0	1	0	0	0
Z_4	—.2	0.1	0	1	0	0
Z_5	—.1	0	—.03	0	1	0
Z_6	—.52	0	0	—3	—2	1

X

1	0	0	0	0	0
.5	0	1	0	0	0
.5	1	0	0	0	0
.25	.1	0	1	0	0
.116	0	.03	0	1	0
1.23	.3	.6	3	2	1

From the steady state condition, we have 1016 patients in residency, i.e., 625 + 391. If the center has a full time psychiatry research group of 3, and a required space for administration, nursing, and reception of 15 rooms. Based on these information the steady state requirements of the proving factor can be determined using parts explosion matrix as follow ;

$(15, 3, 0, 0, 0, 0, 1016)\ (I - Z)^{-1}$

space, psychiatrist, comm., psych, skills, resid.,

(1270 308 , 68 , 3048, 2032, 1016)

to check the accuracy of the estimates, for example :

	Space	Room
Residency	1016/4	254
Skills classes	2032/10	203
Psychiatry	3048/5	609
Psychiatrists	305/2	153
Consultants	68/2	34
Researsch	3/2	2
Administration		15
	Total Rooms	1270

The same way we can check for the number of :

Psychiatrists —	3048/10	305
Research Psychiatrists		3
	Total paych. staff	308
Skills Supervisors	2032/30	68

Parts Explosion Information and Expansion (budgeting)

If the board of the center decided to expand activites so as to increase admission next year by 500 patients, then they can estimate how much funds they need for that purpose just by knowing the requirements for (500 patients) of the providing factors, and the expected prices of next year.

If the increment of admissions would require additional administrative space of 5 rooms, and additional research activities for 2 or more psychiatrists, then the additional requirements of the providing foctors are :

$$(5, 2, 0, 0, 0, 500)\ (I - Z)^{-1}$$

$=$ (623, 152, 30, 1500, 500) of (rooms, psychiatrists, consultants, psychiatry, skills, residency) respectively.

If prices estimated increase $3,000/room per year $30,000/psychiatrist per year, and $27,000/skills consultant a year. The board should raise a total fund of $7,239,000 for fulfillment of the expansion program.

rooms 623 (3000)	1,869,000
psychiatrist 152 (30,000)	4,560,000
skill consul, 30 (27,000)	810,000
Total fund requirements	$7,239,000

IV. Conclusions

The paper utilizes the Markovian analysis and parts explosion matrices in the field of budgeting for service organizations. In service organizations like schools, hospitals, and social welfare programs, services are provided to the recipient during an interval of time, and in different moving stages. These two characteristics of services flows, the duration and moving stages, require different type of information for budgeting. Types of information that are needed include : 1) information regarding the expected number of service recipients in the different stages of the treatment; 2) information regarding the distribution of the total recipients at time (t): 3) information regarding the requirements of each recipient from the providing factors like teachers, rooms, facilities of library and computer, in a case of educational organization.

The Markov steady state, « absorption » matrix helps in answering questions regarding the service recipient population and its distribution over the different stages. The parts explosion matrix provides the rates of requirements of the providing factors. Both the Markov chain and the parts explosion provide the necessary information for budgeting and services flows. The technique can be altered to fit the nonsteady state case. It can be altered to fit program growth, or budget cuts and program reductions.