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التغير الهيكلى ونظرية الكارثة

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التغير الهيكلى ونظرية الكارثة

موجـــز

فى الدراسات الاقتصادية ، عندما يتكون النموذج القياسى من معادلة واحدة فإن القوى المؤثرة على طبيعة مسار المتغيرات الاقتصادية تكون ذات نوعين : هما القـــوى الأساسية Primary forces والقـوى الثانـــوية Secondary forces . وبينما تشكل القوى الأساسية إتجاه المتغير الاقتصادى ، فإن القوى الثانوية تعتبر هى المسئولة عن التقلبات العشوائية حول هذا الاتجاه فاذا كان النموذج القياسى هـو $Y = XB + E$ ، فإن X تمثل متجه القوى الأساسية بينما تمثل E متجه القوى الثانوية ومن أمثلة القوى الأساسية فى مجال الاستيراد على سبيل المثال الأسعار النسبية و / أو الدخول الحقيقية التى تؤثر على الكمية المطلوبة من الواردات . وتضم القوى الثانوية عدداً كبيراً من القوى التى يعتبر إسهام كل منهما ضئيلا ومن أمثلتهـا الإشاعات والتخوفات التى تنتشر فى بيئة تتسم بتضارب وتخبط القرارات الاقتصادية للمسئولين مما يخلق مناخاً اقتصادياً يصعب فيه على المستوردين اتخاذ قرارات سليمة .

والقوى التى أشرنا إليها حالا سواء كانت أساسية أم ثانوية هى قوى محدودة وهى تختلف عن نـوع آخر من القوى ينشأ عن وجوده تغير فى العلاقة الدالية التى كانت تحكم مسار التغير الاقتصادى .

هذا النوع الآخر من القوى يؤتى تأثيره من خلال متجه المعلمات B إذ أنه يغير قيمة ، ويسبب بالتالى ما يسمى بالتغير الهيكلى structural change

ولقد أطلق عليه Rao (1964, p. 180) اسم القوى الدافعة Implusive forces وهو يعرفه بأنه قوى هائلة تعمل لفترة قصيرة جداً وضرب مثالا له بالحرب فعندما تبدأ الحرب وتعلن حالة الطوارىء وتتم تعبئة الاقتصاد تجاه المجهود الحربى يحدث مع مرور الوقت تغير هيكلى للمتغيرات الاقتصادية وهذه القوى لا تأتى فقط من خارج الاقتصاد كما فى حالة الحرب وإنما يمكن أن تأتى من داخله التخفيضات الكبيرة فى القيمة الخارجية للعملة المحلية والتضخم الجامح أو الانكماش الحاد .

والقوى الدافعة هى محور الاهتمام فى هذه الدراسة . وهى ، كما بينا سابقا ، تعكس نفسها على شكل تغير فى متجه المعلمات الذى يستند إليه العلاقة الدالية للمتغير الاقتصادى محل البحث وذلك من فترة إلى أخرى مما قد يؤدى بالتالى إلى تميز سلوك هذا المتغير الاقتصادى بعدم الاستمرارية discontinuity وثنائية القمم bimodality والتشعب divergence . ولتصوير الكيفية التى يمكن أن يتحقق بها مثل هذا السلوك، تستخدم الدراسة أسلوب رياضى تم تطويره حديثاً ويسمى «نظرية الكارثة» Catastrophe Theory _ Catadrophe therory

ونظرية الكارثة نظرية رياضية ظهرت عام ١٩٧٢ م على يد الرياضى الفرنسى الكبير رينيه توم عندما نشر كتابه «الاستقرار البنيوى والتكوين التشكلى» وتطورت بسرعة منذ ذلك الوقت . ومع أن طموح رينيه توم من هذا العمل كان يتمثل فى بناء نماذج رياضية فى علوم البيولوجيا ، فإن الواقع قد عجل باستخدام هذه النظرية فى العلوم الاجتماعية على يد زيمان وتلاميذه . ويعتبر الكثيرون نظرية الكارثة إمتدادا لعلم التفاضل والتكامل ، وإن كانت إمتداداً راديكاليا يتعلق بدراسة طبيعة النقط الحرجة فى المنحنيات الناعمة ودرجة استقرارها وتتميز بقدرتها على وصف التغيرات الفجائية ومن أمثلتها فى العلوم

الطبيعية غليان الماء وذوبان الثلوج ووقوع الزلازل ، ومن أمثلتها فى العلوم الاجتماعية الثورة واندلاع الحرب وانهيار البورصة . ولقد سمى رينيه توم هذه التغيرات المفاجئة بالكارثة لأن هذه الكلمة فى رأيه هى وحدها القادرة على إعطاء الشعور بالتغير المفاجىء أو التغير الدراماتيكى .

والفكرة الأساسية وراء استخدام نظرية الكارثة فى نمذجة الظاهرة محل البحث هى تحويل موقف معين فى العالم الحقيقى إلى شكل هندسى قياسى معروف، واستخدام الخواص الأساسية المعروفة لهذا الشكل لكى نقول شيئاً له معنى عن الظاهرة الأصلية . وليس هذا بالأمر السهل فالأشكال الهندسية متعددة ويعتمد الاختيار بالدرجة الأولى من بين هذه الأشكال على تصور الباحث للشكل الأنسب . وتعرض هذه الورقة أحد نماذج نظرية الكارثة وهو النموذج المعروف باسم نموذج القرن Cusp Catastrophe Model وذلك بالتطبيق على دالة الطلب على الواردات سواء من الناحية البيانية أو من الناحية الرياضية .

STRUCTURAL CHANGE

AND

CATASTROPHE THEORY

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STRUCTURAL CHANGE AND CATASTROPHE THEORY

Introduction :

A single equation model usually states two types of forces that determine the nature of the movement of economic variables. These two forces are what Rao (1964, p. 176) has called them **Primary** and **Secondary** forces. The primary economic forces are those contributing to the trend component in an economic variable. Secondary forces account for random fluctuations around this trend. For example, in a model like Y = XB + E, X represents the vector of primary forces, and E denotes the vector of secondary forces. Secondary forces account for the action of a large number of forces each of whose contribution is negligible. An example for primary forces are changes in relative prices and/or real incomes affecting the quantity demanded of imports. Daily rumours, fears, prejudices are examples of secondary forces that generate an atmosphere in which it becomes difficult for importers to make their decisiono.

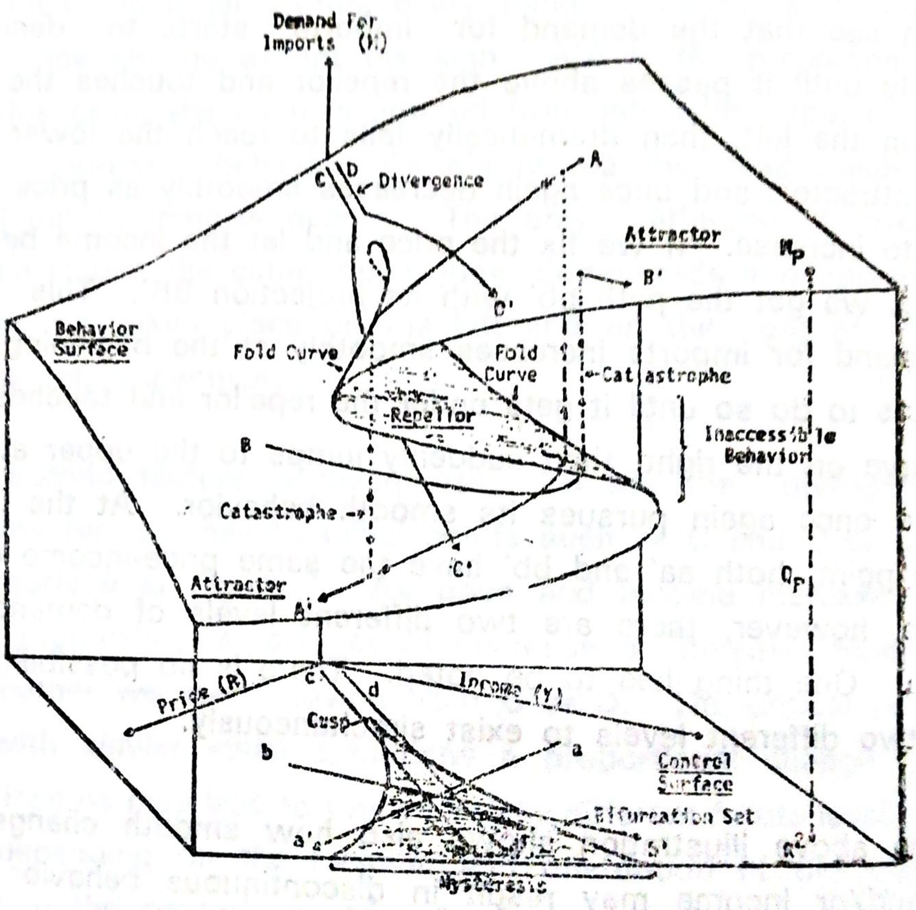
The primary ad secondary forces described so far are finite forces. Sometimes different type of forces may emerge and break the functional relationship previously thought to govern the variable's response by changing the parameter vector, B, underlying the relation. Breifly, such a type may cause structural changes. This type of forces Rao (1964, P. 180) has named it **implusive** forces. Rao defines the implusive force as being a very large force acting for a very short period of time. An example, cited by Rao, is a war. When a war is declared a state of emergency comes into being. The whole economy is geared to the war effort. A structural change in the course over time of the economic variables comes into being. Of course, the implusive forces may come into being from within the economy itself. Large devaluations of domestic currency, and severe (ed-) inflation are just examples.

These implusive forces are the focus for this study. They reflect themselves in changing the parameter vecto underlying the function of an economic variable from time to time and, hence, bring the possibility of the response behavior of that economic variable being discontinuous, bimodal, and divergent. In illustrating how such a behavior could occur a recently developed mathimatical technque called **Catastrophy Theory (CT)** is to be used. A graphical representation of a cusp catastrophe model for import demand will be presented here (see Figure 1 on the next page). The mathimatical counterpart is given in Appendix (A). After a brief discussion of the cusp catastrophe features, we illustrate how impulsive forces over the course of time can change the probability density function of imports, and, thus, by changing the structure create a new parameter vector.

BIMODALITY, DISCONTINUITY, AND DIVERGENCE OF IMPORT BEHAVIOR :

The basic features of the cusp catastrophe model of imports are indicated in Figure I. The graph assumes that implusive forces have twisted the behavior surface of imports creating an area of unstable equilibria called repellor surface (see Appendix (A) for an explanation of being unstable). The line defining the edges of the repellor surface is called the fold curve, and its projection into the control surface is a cusp-shaped curve. Because the cusp marks the boundary where the behavior becomes bimodal, it is called the bifurcation set.

Figure (I)



A Cusp Catastrophe Model For Imports

Given the graph in Figure I, it is clear tha where either price or income is predomenant, there will be just one mode of behavior. However, near the middle of the graph, each point on the control surface has two modeo of behavior : one at a large value and the other at a small value of imports. For example, consider the interesection point of aa' and bb' on the control surface. This point shows how the same price and income combination can correspond to different levels of imports. To see how this could happen, assume income is fixed while price is increasing. This is represented in Figure I by the path aa' on the control surface, which if projectd upto the bevavior Surface gives AA'. From AA' we can see that the demand for imports starts to decrease smoothly until it passes above the repellor and touches the fold curve on the left, then dramatically falls to reach the lower part of the attractor, and once again decreases smoothly as price continues to increase. If we fix the price and let the income be increasing, we get the path bb' with its projection BB'. This time the demand for imports increases smoothly at the beginning and continues to do so until it gets under the repellor and touches the fold curve on the right, then suddenly jumps to the upper attractor, and once again pursues its smooth behavior. At the intersection point, both aa' and bb' have the same price-income combination, however, there are two different levels of demand for imports. One thing has to be noted. There is no possibility for these two different levels to exist simultaneously.

The above illustration shows also how smooth changes in price and/or income may result in discontinuous behavior for imports. On entering the inside of the cusp nothing unusual is observed, but upon further change in the control variable (s), resulting in an exit from the cusp, the system will make a catastrophic jump. The jump phenomenon will occur only when leaving the interior of the cusp from the opposite side to the point of entry.

In addition to the inaccessibility region (repellor surface), bimodality, and catastrophic behavior, the Figure points out to two striking features. The first is hysteresis. A hysteresis effect refers to the phenomenon of the jump from bottom sheet to top sheet being not at the same place as the jump from top sheet to bottom sheet. The hysteresis effect can be demonstrated by examining M, the demand for imports, for, say, fixed income and changing price. Since we have already examined the case for price increase, we move to the other case of price decrease. Assume, with price decreases, we go back along the path aa', i.e., we start at point a' on the control surface and stop at a on the same surface. In other words, every thing is the same for both cases of price change except the sign. A'A is the projection of a'a. In this case, the cusp is entered from left with imports still having a continuous behavior. As a'a leaves the cusp region a positive jump in imports occurs. The only difference between aa' and a'a is that the catastrophic jump downwards and upwards in imports has taken place on the left and on the right of the bifurcation set, respectively.

The second feature is divergence. To see the divergence effect, allow for two nearby initial points such as C and D on the behavior surface in Figure I. As price and income increase by the same proportion, imports could decrease or increase depending on whether we start moving from C or D. The critical point is that, with similar initial conditions a proportional change in Price and income may lead to tundamentally different futute levels of imports, depending on the location and orientation of the bifurcation set in the control space*. As Brown (1979), P. 31) has

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* The proportional increase in price and income is reflected in cc' and dd' on the control surface being straight lines. Note that the conclusion drawn is correct also for nonproportional changes.

pointed out, the location of the bifurcation set is influenced by «the effective force behind the control variables». That is, with both price and income exerting opposite influences upon imports: if the changes in income are of less relative importance than the changes in price, then the bifurcation set would be skewed towards the price axis. Such a situation is very likely if money illusion is assumed to underly the importer's behavior.

DYNAMICS OF STRUCTURAL CHANGES :

The dynamics of import model illustrated in Figure 1 is better be described in terms of the probability density function of imports, f_1. Define a point on the control surface such as $1_0 = (Pr_0, Y_0)$, where each 1 determines a particular probability density function, f_1, of imports magnitude, M. Let f_1 have one or two local maxima. For example, at any point in time when prices are extremely high while real income is very low ordecreasing, a signal is given to importers to reduce their imports and, hence, small imports are very likely (see Figure II at time 6). On the other hand, low prices coupled with rapid growth in real GNP could encourage most importers and a boom is very likely (see Figure II at time 0). Figure I suggests a third possibility. It is probably when both prices and income are at high levels that f_1 becomes double-peaked (see Figure II at time 3).

Now, a particular $I_1 = (Pr_1, Y1)$ will result in either M1 or M'1 (Not shown in any graph) depending on the previous value of M. Had M been closer to M1 than M'1, imports would move to M1 to attain a new maximum. Movement to mi can occur only after the price and income have changed enough so as to unify importers' decisions and force f_1 into a single-peaked distribution. The castastrophic jump to the second local point of maximum at which imports are low (high) will usually occur after the first local maximum of high (low) imports has completely degenerated.

Figure (II)

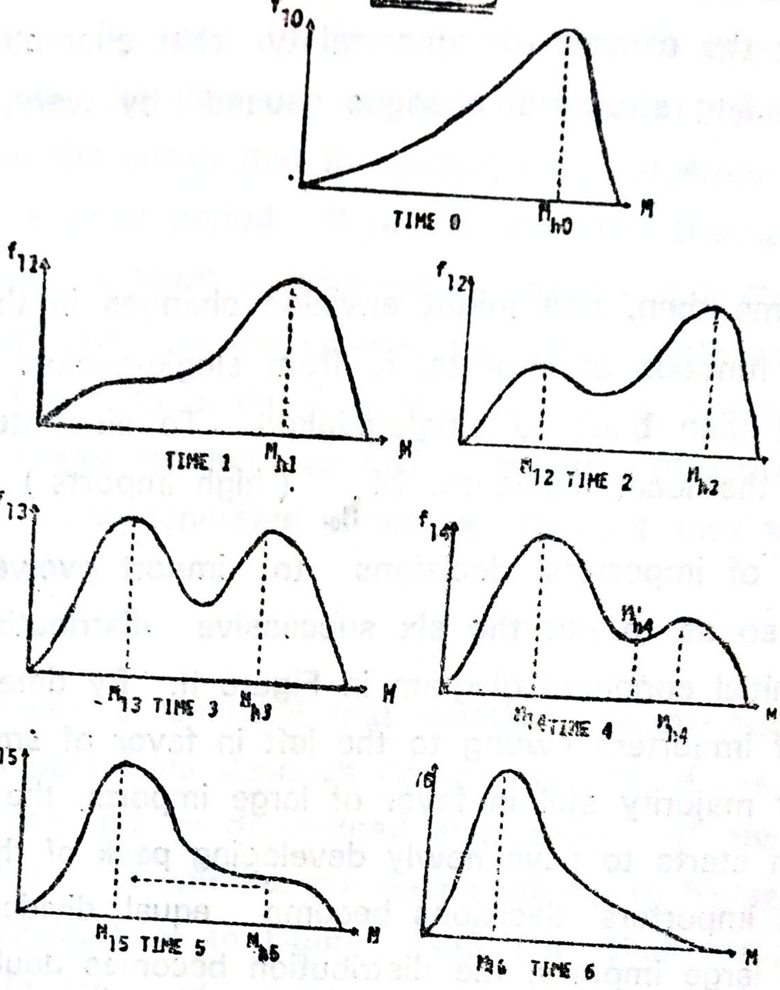
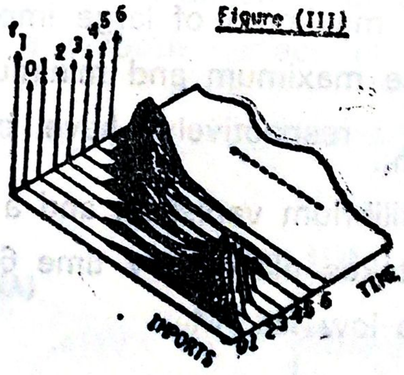


Figure (III)



In CT-terminology, this is called the **Delay Rule** (Zeeman, 1977, p. 313). The delay results from inertia and imperfect information generated by the climate of uncertainty that characterizes the periods preceding structural changes caused by wars, devaluations, etc.

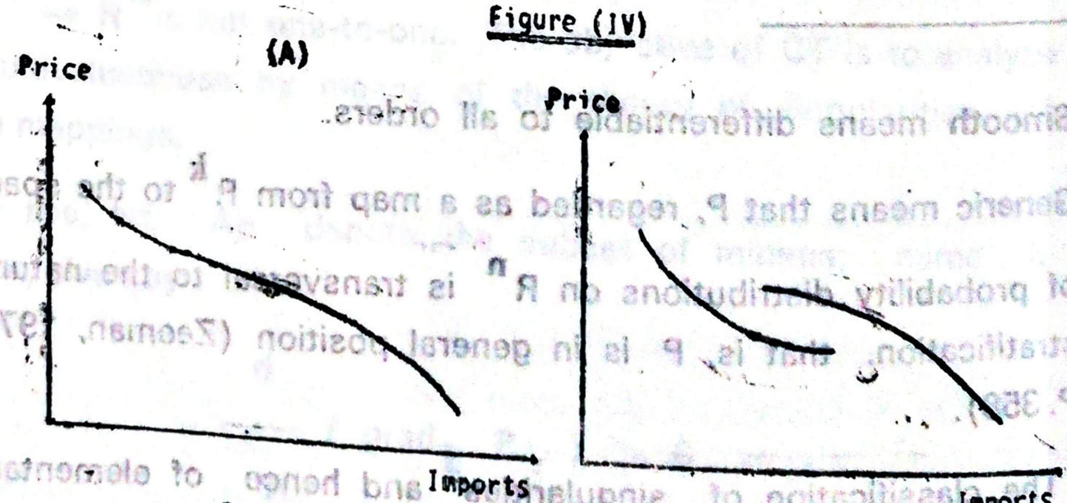
Over time then, one might envision changes in the probability density function of imports, f_1, from single-peaked to double-peaked and then back to single-peaked. To illustrate, suppose imports at the local maximum M_{h_0} (high imports) and let the distribution of importers' decisions to import evolve smoothly with time so as to get the six successive distributions shown with the initial condition diagram in Figure II. By time 2 as the minority of importers swung to the left in favor of small imports while their majority still in favor of large imports, the probability distribution starts to have newly developing peak of the left. By time 3 as importers' decisions become equal divided between small and large imports, the distribution becomes double peaked. By time 4, the old local maximum of large imports starts to degenerate. By time 5, the maximum and minimum points of large imports, M_{h_4} and M_{h_4} respectively, have coalesced at M_{h_5}. A moment later, the equilibrium vanishes, and a catastrophic jump to M_{l_5} (low imports) takes place. By time 6, imports behavior has settled stably in the low field M_{l_6}.

Figure III shows all the distributions superimposed, with the numbers refer to time i, i = 0, .., 6, and the blobs indicating the level of imports in each case.

The preceding discussion can be reillustrated in terms of the more familiar price-uantity diagram. The plane «income = small constant» intersects the attractor surface in Figure I, and gives us the graph of smooth demand curve shown in Figure IV-A. Similarly, the plane «income = large constant» gives the graph in Figure IV-B. But this time, we get two disconnected monotonic curves. As long as the system behaves according to the smooth curve, the demand function of imports can be regarded as stable, in the sense that its underlying parameters are constant throughout a given period. If this is the case, the assuption of constant but unknown parameters in regression analysis is justified. But as soon as the system, for a given high income, starts to behave according to the disconnected curves, the imports function is no longer regarded as stable, because observations about imports are, now, generated under two regimes each has different set of parameters.

The reader is reminded that whether imports have bevaved as Figure I postulates depends on the location of the bifurcation set which can not be determined empirically. All the empirical work can tell us is whether our data confirms to Figure IV-A (no structural change), or to Figure IV-B (the case for structural change). In terms of 3-dimensional diagram, it will tell us whether we have a smooth surface or a twisted surface as in Figure I.

Figure (IV)



$M = M^{C}(Pr|Y{=}small)$ $\qquad$ $M = M^{UC}(Pr|Y{=}large)$

APPENDIX

The purpose of this appendix is to provide a verv brief discussion of the catastrophe theory from the mathimatical point of view. The basic assumption and the main result of the catastrophe theory is first discussed, then a cusp catastrophe model for imports is illustrated.

CATASTROPHE THEORY : BASIC ASSUMPTION AND MAIN RESULT :

The theory of catastrophes has been recently developed by the French mathimatician René Thom in 1972 in an attempt to rationally account for the phenomena of discontinuous change in behavior resulting from a change in parameters of a given model (system).

To start with, let P be a smooth, generic probability function on $R^k \times R^n \rightarrow R$, where R^k is the space of k-control variables, R^n is the space of n-response variables, and R denotes the real numbers*. K is assumed to be $\leqslant 5$, while n is unrestricted*. P

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* Smooth means differentiable to all orders.

Generic means that P, regarded as a map from R^k to the space of probability distributions on R^n is transversal to the natural stratification. that is, P is in general position (Zeeman, 1977, P. 358).

* The classification of singularities and hence of elementary catastrophes goes infinite for $K \geqslant 6$ (Zeeman, 1977, P. 66). Singularity means a point where a vertical line touches the combined graph Mp, i.e., the set of all stationary values of P (Zeeman, 1977, P. 23).

is assumed to represent a dynamical system W. The basic assumption in CT is that this dynamical system attemps to locally minimize P. That is, W is dissipative.

Given any such function P, by fixing the point $Z = R^k$, we obtain a local potential function $P_z : R^n \rightarrow R$, and we may postulate a differential equation describing the gradient dynamical system on R^n.

(1)
$$X = - \operatorname{grad}_x P$$

$$X = - \operatorname{grad}_x P_z$$

$$X = \left(\frac{dp.}{dx} \, .. \, \frac{dp.}{dx_n} \right),$$

$X \in R^n$ Let $Mp \subset R^k \times R^n$ denote the set of all stationary values of P, given by

(2)
$$\operatorname{grade}_x P_z = 0.$$

Thus, the phase trajectory of W will flow towards a minimum of Pz; call it Xz. Xz will usually be multivalued function of z. That is, $Xz : R^k \rightarrow R^n$ is not one-to-one. The objective of CT is to analyze this multivaluedness by means of the theory of singularities of smooth mappings.

To see, let, Ap denote the subset of minima; name it attractor, given by

(3)
$$\frac{d}{dx} (\operatorname{grad}_x P_z) \geqslant 0,$$

then the complement $Gp = Mp - Ap$, call it repellor, is the set of maxima.

That is, Gp is given by

$$(4) \quad \frac{d}{dx}\,(\text{grad}_x\, P_z) < 0^{*}$$

If we let Qp : Mp → Rk be the map induced by the projection of $R^{k+n} \rightarrow R^{k}$** then the classification theorem (Thom, 1972) implies, among other things, that :

(A) Mp (Ap U Gp) is a k-dimensional, smooth without boundaries, generic surface;

(B) When Mp is projected orthogonally onto the control surface, the only singularities that can occur, with $k = 2$, are the fold curve and cusp points***

The importance of Mp being a k-dimensional surface is that Mp is the place where controlling influence is exerted (Costi and Swain, 1975, P. 5). This can be apreciated if we recall that n, the dimension of behavior spece can be very large. By directing our attention to only very few variables, we can easily investigate when and where catastrophic changes occur.

For equations (3) and (4), equation (2) is assumed to hold.

** Qp is known as the catastrophe map.

*** Recall that singularities dpend upon k (see footnote on P.118) The singularities for $k=2$, are given by the equality in equation (3). With $k=4$, the only complete singularities are given by the cusp surface and butterfly points (Zeeman, 1977, P. 343).

A CUSP CATASTROPHE MODEL FOR IMPORTS :

Let us now illustrate the above ideas by considering the cusp catastrophe (whre K = 2, but n still unrestricted), the most important one of seven elementary catastrophes* .

Suppose that the function of demand for imports is given by

(4) $$M = M\ (Pr,\ Y), \qquad Mpr < 0, \quad My > 0,$$

where M is quantity of imports, Pr is relative prics, Y is real income, and Mi is the first derivative with respect to i = Pr, Y. Mi is regarded as the respective elasticities of import demand if M is expressed in long-linear form. Given this function, k = 2, n=1, and control and behavior spaces have coordinates Pr, Y, and M, respectively.

Let $P : R^2 x\ R^1 \rightarrow R$ be given by

(5) $$P\ (Pr,\ Y,\ M) = .25M^4 - .5\ (Y\text{-}Pr)M^2 - (Y+Pr)M^{**}$$

The combined graph (the surface formed by sets of minima and maxima), Mp, is given by

(6) $$\frac{dp}{dM} = M^3 - (Y - Pr)M - (Y+Pr) = 0.$$

The attractor, Ap, is given by the in equality

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* See Zeeman, 1977, P. 27.

** See Zeeman, 1977, P. 27 and P. 332 for a Justification for this form.

$$(7)\quad \frac{d^2p}{dM^2} = 3M^2 - (Y - Pr) > 0.$$

The boundary)Ap of Ap is the fold curve of Mp, and given by

$$(8)\quad \frac{d^2p}{dM^2} = 3M^2 - (Y - Pr) = 0.$$

The projection of 6Ap onto the (Y,Pr)-plane is the bifurcation set, B, (see Figure I). From (8), $M = (3)^{-1/2} (Y - Pr)^{1/2}$ Substituting the equivalent of M into equation (5) gives us the equation of B

$$(9)\quad 27 (Y + Pr)^2 + 4(Y - Pr)^3 = 0.$$

Although the fold curve, dAp is a smooth curve, B has a cusp at the direction of the origin, and that is where the name cusp catastrophe comes from. The fold curve separates the attractor surface, Ap, into two pieces, both of which have dAp as their common boundary. The attractor surface is single-sheeted outside the cusp and is the same as Mp. Over the inside of the cusp the attractor becomes double-sheeted while Mp becomes triple-sheeted. The extra middle sheet being the complement Gp and is given by

$$(10)\quad \frac{d^2p}{dM^2} = 3M^2 - (Y - Pr) < 0.$$

Gp represents a repellor surface, the opposite of an attractor surface. It is the repellor surface that gives the cusp catastrophe model of imports its most interesting features : bimodality, inaçcessiblity, catastrophe, hysteresis, and divergence. The bifurcation set, B, consists of surfaces bounding regions of qualitatively different behavior. Slowly crossing such a boundary may result in a sudden jump in the behavior of imports, giving rise to the term **catastrophe.** The jump is the bifurcaion of he differential equation M = -grad M P, since the basic assumption is that W, the dynamical system, always moves so as to minimize P. This implies that no position can be maintained on the repellor surface which is a set of maxima. As a result W must move from one attractor to another. Hence, although Mp is mathimatically interesting, it is irrelevant from the point of view of the application under consideration because the system stays only on the attractor surface.

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